Financial frictions and stabilization policies *

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Abstract

After the financial crisis of 2007, in most economies carrying out either fiscal consolidations or counter-cyclical fiscal policies, public and private debt have moved in opposite directions, as opposed to pre-2007 evidence. Private deleverage and public debt build-up may affect the recovery path of countries after a recession. In a new Keynesian model with financial frictions, we show that when the economy is hit by a credit risk shock, the negative correlation that arises between public and private debt amplifies the response of GDP. In our setup, the traditional monetary-fiscal policy mix is not enough to offset this *public-private debt mechanism* and therefore bring back economic stability. When macroprudential policy is part of the policy mix, the public-private debt channel can be broken. Interestingly, depending on the macroprudential instrument, a trade-off may arise between private debt and output stabilization.

Keywords: financial accelerator, macroprudential policy, fiscal and monetary policy mix, public and private debt.
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1 Introduction

Since the onset of the financial crisis, different authors[1] have tried to evaluate the macroeconomic policies that some countries implemented in order to restore their pre-crisis levels of GDP growth, inflation or unemployment.[2] The evidence is mixed. On the one hand, fiscal consolidation efforts during the downturn, like those carried out in many European countries, may have deepened the recession, leading to lower private credit and higher public debt. But on the other hand, this negative relationship between private and public debt is also present even if countercyclical fiscal policies were implemented, as could be the case of the U.S.

This paper studies how this link between private and public debt may amplify the business cycle and which policy tools are required to stabilize the economy. Corsetti, Kuester, Meier and Müller (2012) analyze what they call the *sovereign risk channel* through which higher sovereign default risk adversely affects economic activity by raising the financing costs of the private sector. This connection between public and private debt translates, according to these authors, into more volatile business cycles. We claim that not only the cost of financing debt, but also its level matters for economic volatility.

Table 1 presents the correlation between private and public debt (B-D), public debt and output (D-Y) and government spending and output (G-Y), respectively, in the U.S. and Spain for period 1960-2017 (top panel) and the subsample 2007-2017 (bottom panel).[3] It shows that, for the whole sample, the correlation between private and public debt is positive, with government spending being procyclical. However, since the onset of the recent financial crisis and until 2017, both countries present a negative correlation between their levels of private and public debt, with government spending being counter-cyclical in the US and pro-cyclical in Spain. That is, since 1960, recession times have witnessed a build up of both public and private debt, and vice versa. However, this pattern changes during the Great Recession, with public and private debt moving in opposite directions, independently of the cyclicality of government spending. In part, it may be the result of the deleverage process undertaken by the private sector in economies seriously affected by the crisis. In some countries, governments feeling less constrained, enjoyed greater room for cycle stabilization at the cost of more public leverage, for example in the U.S. At the

[1]Among others Gomes and Seoane (2017), and Quint and Rabanal (2014).

[2]Henceforth we will refer to this restitution of the level and growth of the main macroeconomic variables as the "economic recovery".

[3]Variable B in Table 1 includes private debt held by households. Subtracting households' private debt from the series used for the computation of B the correlation between B and D for the subperiod 2007-2017 is -0.511 in the case of Spain and -0.275 in the case of the US.

same time, we also observe countries in which governments were forced to retrench too. In these countries downturns were amplified (GDP volatility increases by almost 40% in Spain during 2007-2017 compared to the sample 1960-2017), with even worse implications for the private sector (e.g. the case of Spain). This may point towards a non-trivial role of fiscal policy in economic stability, displaying this trade-off between public and private debt stabilization.

Table 1: Contemporaneous correlation among main debt and output aggregates, Spain and U.S.

Period 1960-2017			
	$\rho(B,D)$	$\rho(D,Y)$	$\rho(G,Y)$
Spain	0.493	-0.458	0.577
US	0.814	-0.147	0.173
Subperiod 2007-2017			
	$\rho(B,D)$	$\rho(D,Y)$	$\rho(G,Y)$
Spain	-0.556	-0.820	0.672
US	-0.913	-0.333	-0.458

Note: B denotes real private debt-to-real GDP ratio; D is real public debt-to-real GDP ratio; Y represents real GDP, and G is real government consumption. Source: See Appendix A.

Our objective with this paper is twofold. First, we setup a new Keynesian model that accounts for this negative comovement between public and private debt in an economy where fiscal policy aims at stabilizing public debt and with distortionary taxes. All this, in the context of financial frictions in the financing in the private sector, and under alternative characterizations of the monetary policy. The purpose is to assess the ability of standard monetary-fiscal policy mixes to cancel the negative private-public debt correlation that amplifies the business cycle and destabilizes the economy.

The model predicts that in the event of a recession originated in the private sector, output falls. The downturn expands to the rest of the economy, private debt goes down enhancing the fall in investment (financial accelerator) and amplifying the recession. Revenues from tax collection go down, and other things equal, this increases public debt. As a result, during a recession originated in the private sector public and private debt move in the opposite direction, what we call the *private-public debt channel*. This negative feedback between private and public debt acts as an amplifier of the shock on output.

We show that in our framework, fiscal policy is not innocuous. If the government tries to boost output by increasing public consumption, public revenues will experience a direct increase, provided there is a multiplier effect. This may stabilize output with ambiguous effects on public debt stabilization. But if, conversely, the government reacts to the downturn by reducing public consumption to control public debt, output will decrease even more feeding back into falling public revenues and private debt. This trade-off between private and public debt stabilization is even more relevant when financial frictions are at work in the context of a financial recession, and becomes more severe in the presence of automatic stabilizers, contributing to economic instability.

In this framework, we compare different scenarios changing the degree to which monetary and fiscal policies are active or passive, in line with Leeper (1991). We find that standard combinations of active/passive policies do not achieve stabilization of both public and private debt at the same time in response to credit risk shocks. Moreover, these alternative scenarios do not alter the response of the financial accelerator, as we model it.

Given these results, our second objective is to analyze additional tools to mitigate this private-public debt channel, and therefore to smooth business cycle fluctuations. We use macroprudential policy to focus on financially-related variables to stabilize the economy and let fiscal policy focus on public debt stabilization independently of the objective of monetary policy (more or less aggressive on inflation). In the analysis below, we consider standard monetary-fiscal policy mixes.[4]

More concretely, we compare the performance of a macroprudential tool that controls credit growth to the private sector, with an alternative tool that responds to the credit-to-GDP ratio. Once macroprudential policies are at work, we find that the private-public debt channel is partially or completely offset. Our results differ from one scenario to another, what allows us to conclude that the effectiveness of macroprudential measures depends on both the policy mix and the way macroprudential policy is designed.[5] We rank the effectiveness of each policy mix in terms of the implied output and private debt volatility, and by its implications on the private-public debt correlation.

For all the scenarios considered, the macroprudential tool that stabilizes private debt the most after a credit risk shock is the one that reacts to the credit-to-GDP ratio, but this is at the cost of more output instability. In turn, the macroprudential tool that brings more economic stability is the one that reacts to nominal credit growth. The results are robust to alternative parameterization of the macroprudential policy. Therefore, we

[4]Most previous literature has focused on how macroprudential policies interplay with monetary policies. Here, we concentrate on the interaction between fiscal and macroprudential policies given that in some countries traditional monetary policy was either inoperative (at the zero lower bound) or not controlled at the national level (the case of the EMU).

[5]Quint and Rabanal (2014), in an open economy framework, also find that the stabilization effects of macroprudential policies are not symmetric and that depend on the fiscal-monetary policy mix implemented.

conclude that macroprudential policies provide additional tools to eliminate or at least reduce the amplification of business cycles originating in the private sector.

The paper is organized as follows. In Section 2, we review the related literature. Section 3 describes the economy model. In Section 4, we add macroprudential policy to the baseline model. Section 5 then presents the equilibrium and market clearing conditions. Section 6 reports the calibration of the model. In Section 7, we analyze the effects that a credit shock causes in the main economic variables. Section 8 concludes.

2 Related literature

This paper is closely related to the line of research on the analysis of policy mixes using DSGE models. In particular, our paper builds on Fernández-Villaverde (2010) and Gomes and Seoane (2017) who use a new Keynesian framework with financial frictions as in Bernanke, Gertler and Gilchrist (1999).

Fernández-Villaverde (2010) studies the effects of fiscal policy focusing on the use of distortionary taxes and a fiscal rule in the presence of financial frictions. In his model, government spending reacts to changes in the lagged public debt-to-GDP ratio. He finds that when seeking to stabilize output, changes in government spending seem to be more effective than changes in taxes. We build on his model and focus mainly on risk shocks as a key element in the propagation of the recent financial crisis (see for example Christiano et al., 2010). Fernández-Villaverde focuses on the response of output to different fiscal shocks. Going one step further, our analysis emphasizes the role of automatic stabilizers in shaping the response of private and public leverage. We find that the link between these two variables may amplify the effects of financial shocks on output.

The mechanism that we study hinges on the level of public debt being inversely correlated to the level of private debt in the spirit of Corsetti, Kuester, Meier and Müller (2012). These authors analyze the *sovereign risk channel* through which higher sovereign default risk adversely affects economic activity by raising the financing costs of the private sector. Corsetti et al. (2012) argue that offsetting the impact of higher sovereign risk premia requires implementing an expansionary monetary policy (policy rate cuts) and they find that when monetary policy is constrained, the channel operates. We coincide with them in the hypothesis of the existence of a channel that connects public debt with the private sector, and we look into an alternative instrument that could offset the mechanism, that is not monetary policy: macroprudential policy.

Our paper is also closely related to Gomes and Seoane (2017). These authors argue that different combinations of active/passive monetary and fiscal policies are able to explain the different recovery paths across countries, summarized in their paper by the US and Euro Area (henceforth EA), after the Great Recession. We differ from their model in the use of distortionary taxation instead of lump-sum taxes. This feature turns out to be crucial in our analysis, since the presence of automatic stabilizers is key for the propagation of financial shocks to the public sector and for the transmission of macroprudential measures from the private sector to the rest of the economy. We compare the effects of the private-public debt mechanism under an active fiscal policy scenario (what would have been the US case) to the effects under a passive fiscal policy scenario (what could be referred to as the case of Spain).[6] We find that the traditional monetary-fiscal policy mix is not enough to stabilize the economy after a credit risk shock. The fact that monetary policy does not react to financial variables calls for additional tools that support and coordinate with fiscal policy to enhance economic stability.

We consider an additional instrument for macroeconomic stabilization: macroprudential policies, and contribute to the literature by studying its interaction with alternative fiscal-monetary policy mixes in a model with financial frictions. There is an extensive literature that analyzes the interaction between monetary and macroprudential policies.[7] The former are aimed at price stability, while the latter are aimed at financial stability. In line with our framework, Rubio and Carrasco-Gallego (2014) implement a DSGE model that combines monetary and macroprudential instruments to evaluate their effects on business cycles, welfare and financial stability. They find that the restriction of credit during booms contributes to business cycle stabilization and improves welfare, while it might enter in conflict with monetary policy. However the stability of the system is improved when both policies are coordinated.

Quint and Rabanal (2014) study the effects of a negative risk shock in a two-country currency union. These authors observe that monetary policy by itself cannot contain the accelerator effects of the economy, so they introduce a macroprudential policy that constrains either the credit-to-GDP ratio or the nominal credit growth and delivers stability. We incorporate their macroprudential tools to our closed-economy analysis and study the

[6]We do not consider the case of the EA as in Gomes and Seoane (2017) because even though it is easy to characterize monetary policy at the EA level, it is more difficult to study a common fiscal policy in this scenario. That is, when analyzing the EA, imposing an active/passive fiscal policy may not be representative of the environment it pretends to study. In our paper, we focus on the analysis of the effects of the policy mix on the main variables of the financial sector of an individual closed economy. See Malmierca (2019) for the analysis of the policy mix in a currency union.

[7]The literature has grown considerably in the recent years. Just to cite some of the papers Gerlach et al. (2009), Angelini et al. (2012), de Paoli and Paustian (2013), Gelain and Ilbas (2017).

interactions with fiscal policy.

Despite the extensive literature on the interaction of macroprudential and monetary policies, there is not much about how to coordinate macroprudential and fiscal policies. This paper tries to fill this gap. Claessens (2014) not only reviews the interaction of macroprudential instruments with monetary policy measures but also mentions the importance of coordinating macroprudential actions with other policies, such as fiscal or microprudential. Regarding fiscal policy, some tax policies can contribute to systemic risk by encouraging private leverage (for instance, when interest payments are tax deductable) and therefore macroprudential authorities need to coordinate with fiscal authorities (Claessens, 2014). The present paper sheds new light on policy mix coordination: we show that standard monetary and fiscal policy combinations are not enough to stabilize both public and private debt, and propose the inclusion of macroprudential instruments as part of the policy mix.

One important contribution of this paper is the analysis of the interaction between fiscal and macroprudential policies. Fiscal policy becomes relevant in our analysis due to the role of distortionary taxes. For example, in the presence of the *public-private debt mechanism*, a falling private debt that depresses the economy may exacerbate the fall in tax collection, leading to further adjustments in government expenditure and higher public debt. The use of a policy directed to stabilize private debt may reduce this amplification mechanism by letting fiscal policy focus on public finances stabilization.

3 The model

Our model economy follows closely Fernández-Villaverde (2010), which incorporates the financial accelerator as in Bernanke, Gertler and Gilchrist (1999). The economy is composed of households, intermediate good producers, final good producers, entrepreneurs, capital goods producers, financial intermediaries and a national government that sets fiscal, monetary and macroprudential policies. We quickly present the model and focus on the introduction of macroprudential policies.

3.1 Households

There is a continuum of households with infinite life. The representative household maximizes his utility function, choosing consumption, c_t, time devoted to work, l_t, and financial assets composed of deposits, a_t, and government bonds, d_t, both in positive amounts. The individual's utility function is given by

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\log\left(c_t - hc_{t-1}\right) - \psi \frac{l_t^{1+\vartheta}}{1+\vartheta} \right], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor; $h \geq 0$ reflects the degree of habit persistence; $\psi > 0$ denotes the magnitude of the labor disutility relative to consumption utility; and $\vartheta > 0$ is the inverse of the Frisch elasticity of labor supply.

The household makes decisions subject to the following budget constraint:

$$\begin{aligned}(1+\tau_{c,t})\, c_t + \frac{a_t}{p_t} + \frac{d_t}{p_t} = (1-\tau_{l,t})\, w_t l_t + \left[1 + (1-\tau_{R,t})\,(R_{t-1}-1)\right] \frac{a_{t-1}}{p_t} \\ + R_{t-1}^d \frac{d_{t-1}}{p_t} + T_t + F_t + tre_t.\end{aligned} \tag{2}$$

The left hand side of equation (2) represents the household's expenditures in real terms. The right hand side describes the sources of income to the household: labor income, $w_t l_t$, where w_t is the real wage; interests on last period investment on deposits, $R_{t-1}a_{t-1}$ and on public assets, $R_{t-1}^d d_{t-1}$; and net transfers that households receive from the government, T_t. The model includes distortionary taxes on real consumption, $\tau_{c,t}$, on labor income, $\tau_{l,t}$ and on net returns on deposits, $\tau_{R,t}$.[8] Dividends are paid by firms to households, F_t; and households receive from entrepreneurs a net transfer, tre_t, defined as follows:

$$tre_t = (1-\gamma^e)\, n_t - w^e. \tag{3}$$

As will be explained in detail below, γ^e regulates the rate of entrepreneurs that survives from one period to the next one. Then the net wealth of the dead entrepreneurs, $(1-\gamma^e)\, n_t$, is paid back to households and these transfer w^e to incoming entrepreneurs, that is the initial real net wealth of the new entrepreneurs.

The first order conditions obtained from the representative household's problem are

$$\frac{1}{c_t - hc_{t-1}} - \beta E_t \frac{h}{c_{t+1} - hc_t} = \lambda_t \left(1 + \tau_{c,t}\right), \tag{4}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{\left[1 + (1-\tau_{R,t+1})\,(R_t - 1)\right]}{\Pi_{t+1}}, \tag{5}$$

[8]Returns on sovereign debt are not taxed because, as Fernández-Villaverde (2010) says, otherwise the government would have to pay a higher interest rate on public debt to compensate for the lower net return that households would receive due to the tax, thus the effect would be the same.

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_t^d}{\Pi_{t+1}}, \tag{6}$$

$$\psi l_t^{\vartheta} = (1 - \tau_{l,t})\, w_t \lambda_t, \tag{7}$$

where λ_t is the Lagrange multiplier that represents the marginal value of wealth of households, and Π_t denotes the gross inflation rate.

3.2 Intermediate goods producers

These agents produce differentiated goods that are then sold in a monopolistic market to final good producers, who use them in their production process. Each intermediate good producer, i, chooses labor l_{it} and capital k_{it-1} as factors of production and creates output y_{it} through the following constant returns to scale Cobb-Douglas production function:

$$y_{it} = e^{z_t} k_{it-1}^{\alpha} l_{it}^{1-\alpha}, \tag{8}$$

where $0 \leq \alpha \leq 1$ is the capital share of the intermediate production function.

Technology follows an exogenous AR(1) process $z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}$, where $0 < \rho_z < 1$, and $\varepsilon_{z,t} \backsim N(0,1)$, being ρ_z the persistence coefficient, and σ_z the volatility of the technology shock.

Labor is rented from households in exchange for real wages w_t and capital from entrepreneurs (whose problem is explained below) in exchange for a rental real interest rate r_t. Cost minimization implies

$$k_{it-1} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t} l_{it}. \tag{9}$$

These firms reset their prices through a Calvo pricing mechanism by which, each period, a fraction $1-\theta$ of them can choose to change their price, while a fraction θ of firms have to keep the previous period price which is then indexed to past inflation.

Firms resetting their price in period t maximize the following expression:[9]

$$E_t \sum_{\tau=0}^{\infty} (\beta\theta)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left[\left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi}}{\Pi_{t+s}} \frac{p_{it}}{p_t} - mc_{t+\tau} \right) y_{it+\tau} \right], \tag{10}$$

[9]The expression represents the discounted sum of the difference between the optimizing firm's revenues and its marginal cost, that is, the discounted profits.

subject to a sequence of demand functions given by the final good producer

$$y_{it+\tau} = \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi}}{\Pi_{t+s}} \frac{p_{it}}{p_t} \right)^{-\varepsilon} y_{t+\tau}, \tag{11}$$

for $\tau = 0, 1, 2, ...$ where the marginal value of wealth of households/stochastic discount factor, $\frac{\lambda_{t+\tau}}{\lambda_t}$ is taken as given by the monopolistic firm; mc_t denotes the marginal cost of the intermediate good producer; p_{it} is the price set in period t by firm i; p_t is the aggregate price level; $\Pi_t = \frac{p_t}{p_{t-1}}$ denotes gross inflation; $\chi \in [0, 1]$ represents the degree of price indexation; $y_{it+\tau}$ denotes output in period $t+\tau$ for a firm that last reset its price in period t; $y_{t+\tau}$ is the aggregate level of output in time $t+\tau$; and $\varepsilon \geq 1$ is the elasticity of substitution across goods. Let the reset price relative to the price level be $\Pi_t^* = \frac{p_t^*}{p_t}$.

So the first order conditions for these intermediate firms are:[10]

$$\frac{k_{t-1}}{l_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t}, \tag{12}$$

$$mc_t = \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} \frac{w_t^{1-\alpha} r_t^{\alpha}}{e^{z_t}}, \tag{13}$$

$$\varepsilon f_t^1 = (\varepsilon - 1) f_t^2, \tag{14}$$

where

$$f_t^1 = \lambda_t mc_t y_t + \beta\theta E_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}} \right)^{-\varepsilon} f_{t+1}^1, \tag{15}$$

and

$$f_t^2 = \lambda_t \Pi_t^* y_t + \beta\theta E_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}} \right)^{1-\varepsilon} f_{t+1}^2 \left(\frac{\Pi_t^*}{\Pi_{t+1}^*} \right), \tag{16}$$

where, following Fernández Villaverde (2010), f_t^1 and f_t^2 are two auxiliary variables.

The aggregate price index is given by

$$1 = \theta \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t} \right)^{1-\varepsilon} + (1-\theta) \, \Pi_t^{*(1-\varepsilon)}. \tag{17}$$

[10] Since all intermediate good producers face the same prices and because of market clearing, subscript i can be removed from the previous expression, meaning all the monopolistic producers choose the same ratio for the production factors they use $\frac{k_{it-1}}{l_{it}}$ and so, henceforth, capital and labor will be expressed in aggregate levels.

3.3 Final goods producers

Final goods producers buy intermediate goods from intermediate goods producers and combine them to obtain the homogeneous final good according to the following Dixit-Stiglitz technology function:

$$y_t = \left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{18}$$

where y_t is the aggregate demand of the economy, and ε is the elasticity of substitution across goods. The final good is sold to consumers in a perfect competitive market. These firms maximize profits taking both the price of the intermediate good p_{it} and the price of the final good p_t as given. The price level is given by

$$p_t = \left(\int_0^1 p_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}. \tag{19}$$

3.4 Capital goods producers

These agents operate in a perfectly competitive market and create new capital, x_{t+1} using investment, i_t, and installed capital, x_t, via the following production function:

$$x_{t+1} = x_t + \left(1 - S \left[\frac{i_t}{i_{t-1}} \right] \right) i_t, \tag{20}$$

where $S\left[\frac{i_t}{i_{t-1}}\right]$ denotes adjustment costs, such that $S'[\cdot] > 0; S''[\cdot] > 0; S[1] = 0$; and $S'[1] = 0$.

Installed capital is previously purchased from entrepreneurs. Let q_t denote the relative price of capital, then discounted profits are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[q_t \left(1 - S \left[\frac{i_t}{i_{t-1}} \right] \right) i_t - i_t \right] = (1-\delta) k_{t-1}, \tag{21}$$

where $\delta \in [0, 1]$ is the capital depreciation rate, and $x_t = (1-\delta) k_{t-1}$. The first order condition is the following:

$$q_t \left(1 - S \left[\frac{i_t}{i_{t-1}} \right] - S' \left[\frac{i_t}{i_{t-1}} \right] \frac{i_t}{i_{t-1}} \right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S' \left[\frac{i_{t+1}}{i_t} \right] \left[\frac{i_{t+1}}{i_t} \right]^2 = 1. \tag{22}$$

Notice that absent adjustment costs $q_t = 1$. And the law of motion of capital is expressed by

$$k_t = (1-\delta)k_{t-1} + \left(1 - S\left[\frac{i_t}{i_{t-1}}\right]\right) i_t. \quad (23)$$

3.5 Entrepreneurs

Entrepreneurs are in charge of transforming installed capital, x_t, into inputs for use by intermediate goods producers, k_{t-1}. Each period, entrepreneurs buy new capital, k_t, from capital goods producers at a price q_t, to undertake their investment. Their output is then rented to intermediate goods producers at a cost r_{t+1} per unit of capital rented.

Let r_{t+1} be the price that the entrepreneur charges to the intermediate goods producer for the rental of k_t, and let $q_{t+1}(1-\delta)$ be the cost that the capital goods producer assumes for the repurchase of the old non-depreciated capital, paid to the entrepreneur at the end of the period. We can then define the ex-post average return of the entrepreneur per unit of investment between t and $t+1$, R^k_{t+1}, as

$$R^k_{t+1} = \Pi_{t+1}\frac{r_{t+1} + q_{t+1}(1-\delta)}{q_t}. \quad (24)$$

Their technology is affected by an idiosyncratic shock, ω_{t+1}, that is lognormally distributed with a cumulative distribution function represented by $F(\omega, \sigma_{\omega,t})$ with parameters $\mu_{\omega,t}$ and $\sigma_{\omega,t}$ such that $E_t\omega_{t+1} = 1$ for all t. The distribution function $F(\omega, \sigma_{\omega,t})$ denotes the probability of default.

For the purpose of our analysis it is important to consider the evolution of the standard deviation as it represents the credit risk of our model. The dispersion follows:[11]

$$\widehat{\sigma}_{\omega,t} = \rho_{\sigma_\omega}\widehat{\sigma}_{\omega,t-1} + \eta_{\sigma_\omega}\varepsilon_{\sigma_\omega,t} \text{ where } \varepsilon_{\sigma_\omega,t} \backsim N(0,1), \quad (25)$$

where ρ_{σ_ω} is the persistence coefficient that takes values within [0,1] and η_{σ_ω} is the volatility of the shock. The shock $\varepsilon_{\sigma_\omega,t} \backsim N(0,1)$ is revealed at the end of the period, just before the investment decisions for $t+1$ are taken.

Entrepreneurs use both internal and external funds for the purchase of the new installed capital. Internal funds are composed by the end-of-period net worth (or equity of the entrepreneurs), n_t; while external funds consist of loans (or liabilities of the entrepreneurs) borrowed from financial intermediaries, b_t. Therefore the amount they

[11] We use the notation $\widehat{x}_t$ to refer to the log-linearized version of variable x_t and $\overline{x}$ for the steady state value of the same variable.

borrow is given by[12]

$$\frac{b_t}{p_t} = q_t k_t - n_t. \tag{26}$$

The realization of ω_{t+1} is private information to entrepreneurs, and the contract with financial intermediaries is signed before it is known. This private information leads to a possible moral hazard problem that is solved via a standard debt contract.

3.5.1 Costly state verification problem

As in Bernanke, Gertler and Gilchrist (1999), we consider a costly state verification (CSV) problem: entrepreneurs observe their outcome for free, but financial intermediaries need to pay a cost, proportional to the gross payoff of the entrepreneur's capital.

At the moment of the debt contract agreement, there is aggregate uncertainty because R^k_{t+1} is not known yet.[13] The entrepreneur decides on the amount of capital he wants to purchase, that is, his expenditures for period t, $q_t k_t$, and therefore the amount of external funds that he needs, $\frac{b_t}{p_t}$.

The contract will establish a state-contingent non-default repayment R^l_{t+1} (dependent on the ex-post realization of R^k_{t+1}) that the entrepreneur promises to pay to the financial intermediary in case he succeeds in his investment project. The standard debt contract also specifies a state-contingent threshold value of the idiosyncratic shock ϖ_{t+1} (dependent on the ex-post realization of R^k_{t+1}), below which the entrepreneur defaults. The threshold is determined by the following condition:

$$R^l_{t+1} b_t = \varpi_{t+1} R^k_{t+1} p_t q_t k_t. \tag{27}$$

Both R^l_{t+1} and ϖ_{t+1} are chosen to maximize the entrepreneur's return and such that it is worth for the financial intermediary to enter into the contract, that is,

$$\left[1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] R^l_{t+1} b_t + (1-\mu) \int_0^{\varpi_{t+1}} \omega dF\left(\omega, \sigma_{\omega,t}\right) R^k_{t+1} p_t q_t k_t = R_t a_t, \tag{28}$$

which states that the financial intermediary must be at least indifferent between lending to entrepreneurs or getting the safe interest rate on loanable assets.

[12]This expression means that the contract is set in nominal terms.

[13]We briefly describe the case of aggregate uncertainty. A detailed explanation can be found in Appendix B.

Finally, the average net wealth[14] is

$$n_t = \gamma^e \frac{1}{\Pi_t} \left\{ \left[1 - \mu G\left(\varpi_t, \sigma_{\omega,t-1}\right)\right] R_t^k q_{t-1} k_{t-1} - R_{t-1} \frac{b_{t-1}}{p_{t-1}} \right\} + w^e. \tag{29}$$

The standard debt contract is solved by maximizing the entrepreneur's expected returns

$$\int_{\varpi_{t+1}}^{\infty} \omega dF\left(\omega, \sigma_{\omega,t}\right) R_{t+1}^k p_t q_t k_t - \left[1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] \varpi_{t+1} R_{t+1}^k p_t q_t k_t, \tag{30}$$

subject to the participation constraint of the financial intermediary, equation (28).

3.6 Financial Intermediaries

In our model financial intermediaries receive deposits from households, a_t, and make loans to entrepreneurs, b_t. Financial intermediaries operate in a perfectly competitive market. Their objective function is given by

$$\left\{ \left[1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] R_{t+1}^l b_t + (1-\mu) \int_0^{\varpi_{t+1}} \omega dF\left(\omega, \sigma_{\omega,t}\right) R_{t+1}^k p_t q_t k_t - R_t a_t \right\}, \tag{31}$$

which shows expected returns in case of a successful project, plus revenues in case of default, minus the costs in terms of deposits for the financial intermediary.

3.7 Government

In this model, the government sets monetary and fiscal policy, and when considered, also macroprudential policy.

3.7.1 Fiscal policy

There is a government setting taxes, government spending and public debt, subject to the following budget constraint:

$$\frac{d_t}{p_t} = g_t + R_{t-1}^d \frac{d_{t-1}}{p_t} - tax_t, \tag{32}$$

[14]Average net wealth equals the wealth of the individual entrepreneur since it can be shown that all the entrepreneurs get the same leverage ratio.

where d_t denotes current issue of nominal public debt, and g_t is government spending. Finally, tax_t denotes tax revenues defined by

$$tax_t = \tau_{c,t} c_t + \tau_{l,t} w_t l_t + \tau_{R,t} \left(R_{t-1} - 1\right) \frac{a_{t-1}}{p_t}. \tag{33}$$

In the analysis below, tax rates will be considered time-invariant.

Following Fernández Villaverde (2010), we assume that government spending evolves by the following fiscal rule:

$$\widehat{g}_t = \gamma_g \widehat{g}_{t-1} + d_g \frac{d_{t-1}}{p_t y_t} + d_y \widehat{y}_{t-1} + \sigma_g \varepsilon_{g,t}, \tag{34}$$

where $d_g \leq 0$ is the sensitivity of government expenditure to changes in the debt-to-GDP ratio, its sign reflects the objective of public debt stabilization; and $d_y \leq 0$ to denote the countercyclicality of fiscal policy.

3.7.2 Monetary policy

Government in this model is also in charge of monetary policy. To this end, he uses the nominal interest rate as the monetary policy instrument. It sets monetary policy according to a Taylor rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi} \left(\frac{y_t}{y}\right)^{\gamma_y}\right]^{(1-\gamma_R)} \exp(\sigma_m m_t), \tag{35}$$

where $\gamma_R \in [0, 1]$ is the persistence parameter; $\gamma_\Pi \geq 0$ and $\gamma_y \geq 0$ indicate how strong is the response of the interest policy rate to deviations of Π_t and y_t from their steady states, respectively; and σ_m is the volatility of the monetary policy shock, m_t. The nominal interest rate is modified through open market operations financed by transfers, T_t.

4 Macroprudential policy

The main difference of our model with respect to Fernández-Villaverde (2010) is that we include a macroprudential authority that conducts policies to control the volatility and growth of private debt. Through macroprudential instruments we aim at stabilizing private debt volatility in order to guarantee a more stable cycle, so that the private-public debt channel is offset and with it, the amplification mechanism that it has in the economy.

We focus on imposing macroprudential restrictions on the banking system, that is, on financial intermediaries' balance sheet, by establishing a minimum level of deposits that banks must have on hand as cash or safe assets, limiting the funds to make loans to borrowers.[15] This measure would be equivalent to a reserve requirement ratio.

Following Quint and Rabanal (2014), we insert a macroprudential tool in the model economy by limiting the funds available to lend from the financial intermediaries in the following way:

$$\frac{1}{\eta_t} a_t = b_t, \tag{36}$$

where η_t is a new variable that reflects the credit market conditions. The macroprudential regulation will affect financial variables counter-cyclically. Higher values of η_t reflect a tightening of macroprudential policy, while lower values reflect an easing of macroprudential policy. This macroprudential rule implies that, when the regulation is tightening, financial intermediaries can only lend a fraction of the deposits that households invest.[16] Following Quint and Rabanal (2014) we also make η_t dependent on credit market conditions, Ψ_t, as follows:

$$\eta_t = (\Psi_t)^{\gamma_\eta}, \tag{37}$$

taking γ_η as a parameter that indicates the degree of responsiveness of η_t to the credit market conditions considered. If $\gamma_\eta = 0$ then the macroprudential variable is $\eta_t = 1$ so there would not be macroprudential regulation in the model. We will consider two alternative macroprudential instruments. First, we define Ψ_t as the nominal private credit growth, that is,

$$\Psi_t = \frac{\bar{b}_t}{\bar{b}_{t-1}} \Pi_t, \tag{38}$$

where $\bar{b}_t = \frac{b_t}{p_t}$. This means that the macroprudential instrument becomes tightening as nominal private credit in the current period grows with respect to nominal private credit in the previous period.

Second, we consider Ψ_t as the private credit-to-GDP ratio, that is,

$$\Psi_t = \frac{\bar{b}_t}{y_t}. \tag{39}$$

[15] Claessens (2014) classifies the whole set of macroprudential instruments into 5 different categories: restrictions on borrowers (LTV and DTI ratios), capital and provisioning requirements, other restrictions on financial institutions' balance sheets, taxations and levies on activities or balance sheets and other institutional-oriented measures.

[16] However, in line with Quint and Rabanal (2014) we also allow the macroprudential instrument to behave symmetrically and go below one. The macroprudential rule implies that, when the regulation is easing, the monetary authority will provide liquidity to financial intermediaries so that they can lend more funds than the amount of deposits they hold on their balance sheet.

In this case, the macroprudential instrument becomes tightening when there is an increase of the private credit-to-GDP ratio.[17]

The introduction of macroprudential policies affects the credit conditions in our model.[18] In particular, the lending-deposit spread is positively affected,

$$\frac{R^l_{t+1}}{R_t} = \frac{\eta_t}{\left[1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] + \frac{(1-\mu)}{\varpi_{t+1}} \int_0^{\varpi_{t+1}} \omega dF\left(\omega, \sigma_{\omega,t}\right)}. \tag{40}$$

Notice that when the macroprudential policy is tightening ($\eta_t > 1$), the lending-deposit spread increases, while when the macroprudential policy is easing the lending-deposit spread goes down. That is, a tightening of macroprudential policy means less funds are available to lend without any change in the policy rate. This increases the gap between lending and deposits.

The one period interest rate of the loan, R^l_{t+1}, is set on the contract that the financial intermediary agrees with the entrepreneur, together with ϖ_{t+1}. The previous expression shows that R^l_{t+1} also depends on the level of η_t for the current period so the macroprudential policy affects the contractual agreement. Therefore, when the macroprudential rule is too restrictive the R^l_{t+1} set in the contract is higher than in the case in which macroprudential policy is relaxed. This ensures that when we introduce macroprudential policy, even if part of the households' deposits are not borrowed by the entrepreneurs, financial intermediaries can still obtain zero profits paying the same and not a lower R_t to households for the deposits they hold because the loan rate, R^l_{t+1}, is increased. Therefore, despite macroprudential policy, lending funds in the form of deposits to financial intermediaries is still worth it for households. Entrepreneurs, however, support a higher cost of debt if they need to borrow when macroprudential policy is tightening, and a lower cost of debt if they need to borrow when macroprudential policy is easing. As a consequence private credit is affected not only from the supply side but also from the demand side, which is the goal of the macroprudential authority.

[17] We assume $\eta = 1$ in steady state because, as in Quint and Rabanal, the macroprudential instrument only reacts to deviations of Ψ_t from steady state values.

[18] A detailed explanation can be found in Appendix C.

5 Aggregation and Equilibrium

Aggregate output in the model is given by

$$y_t = c_t + i_t + g_t + \mu G\left(\varpi_t, \sigma_{\omega,t-1}\right)\left(r_t + q_t\left(1-\delta\right)\right) k_{t-1}, \tag{41}$$

from the demand side. And the aggregate supply is

$$y_t = \frac{1}{\upsilon_t} e^{z_t} k_{t-1}^{\alpha} l_t^{1-\alpha}, \tag{42}$$

being υ_t the inefficiency created by price dispersion, that evolves as:

$$\upsilon_t = \theta \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\varepsilon} \upsilon_{t-1} + (1-\theta)\left(\Pi_t^*\right)^{-\varepsilon}. \tag{43}$$

The equilibrium in this model can be defined as the sequence of quantities $\{c_t,\ l_t,\ a_t,\ k_t,\ i_t,\ b_t\}_{t=0}^{\infty}$; fiscal policy $\{g_t,\ tax_t,\ d_t\}_{t=0}^{\infty}$; prices $\{r_t,\ w_t,\ q_t\}_{t=0}^{\infty}$, and interest rates $\{R_t^d,\ R_t,\ R_t^k,\ R_t^l\}_{t=0}^{\infty}$, given exogenous variables $\{z_t,\ \widehat{\sigma}_{\omega,t}\}_{t=0}^{\infty}$ such that:

- optimization problems for all agents in the model are satisfied;
- all markets clear, that is,

$$y_t = c_t + i_t + g_t + \mu G\left(\varpi_t, \sigma_{\omega,t}\right)\left(r_t + q_t\left(1-\delta\right)\right) k_{t-1},$$

$$y_t = \frac{1}{\upsilon_t} e^{z_t} k_{t-1}^{\alpha} l_t^{1-\alpha},$$

$$l_t^s = l_t^d,$$

$$x_t = (1-\delta) k_{t-1},$$

$$\begin{cases} a_t = b_t & \text{if macroprudential policy is not included,} \\ \frac{1}{\eta_t} a_t = b_t & \text{if macroprudential policy is included;.} \end{cases}$$

- plus the laws of motion

$$k_t = (1-\delta) k_{t-1} + \left(1 - S\left[\frac{i_t}{i_{t-1}}\right]\right) i_t, \text{and}$$

$$\frac{d_t}{p_t} = g_t + R_{t-1}^d \frac{d_{t-1}}{p_t} - tax_t.$$

6 Calibration of the parameters and steady state

The model is log-linearized around the non-stochastic steady state, and simulated to exogenous shocks.

Table 2 shows the parametrization we use in our model. We calibrate most of the parameters based on Gomes and Seoane (2017), Fernández-Villaverde (2012), Fernández-Villaverde (2010) and Bernanke, Gertler and Gilchrist (1999).

Preferences. We set the discount factor to $\beta = 0.999$ and $\Pi = 1.005$ what imply an average annual real interest rate equal to 0.4%; habits on consumption are $h = 0.5$, and the Frisch elasticity of labor is $1/\vartheta = 2$. Labor in steady state is $l = \frac{1}{3}$.

Technology. The capital share, α, is set equal to 0.33; capital depreciation rate, δ, equals 8.9% at an annual rate; and capital adjustment costs are such that $S^{"}[1] = 14.477$. The Calvo pricing parameter, θ, is 0.8 what means on average 5 quarters of duration of prices; the degree of indexation to past inflation, χ, equals 0.6; and the elasticity of substitution across goods, $\varepsilon = 8.577$, what implies a markup of around 13% in the goods sector.

Financial variables. We consider monitoring costs, μ, are 15% of the entrepreneur's output; the loan-to-capital ratio is set equal to $\frac{\bar{b}}{k} = \frac{1}{3}$; the survival rate of entrepreneurs is $\gamma^e = 0.975$ and the annual probability of default is assumed to be 3%.

Fiscal policy. The steady state values for tax rates are taken from Fernández-Villaverde (2010) and equal to $\tau_l = 0.24$, $\tau_r = 0.32$; government spending-to-GDP ratio equals 20%, and the debt-to-GDP ratio is 60%. Given these values τ_c is determined from the government's budget constraint. Parameters d_g and d_y depend on the active or passive fiscal policies that characterize each scenario considered.

Monetary policy. In our analysis below, monetary policy covers different scenarios, mainly active and passive policies, depending on the strength of the response to inflation deviations from target.

Macroprudential policy. The macroprudential policy parameter, γ_η, is set to 1.75 in all the scenarios considered.

Shock processes. We set autoregressive coefficients equal to 0.95, and standard deviations are taken from the empirical evidence and past literature, as summarized in Table 2.

7 Impulse Response Functions (IRFs)

Following Leeper's characterization of fiscal and monetary policies, we structure the analysis in two different scenarios according to either monetary or fiscal dominance. For fiscal policy to be passive we employ a fiscal rule that aims at stabilizing public leverage, and for fiscal policy to be active we implement a fiscal rule that reacts relatively more to fluctuations in GDP than to changes in government debt.[19] These two specifications seem to be consistent with the empirical evidence for Spain (Boscá et al., 2017) and the U.S. (Davig, 2018) during the Great Recession, as will become clear below.

To analyze the implications of the private-public debt mechanism, we analyze two possible scenarios. Firstly, a scenario in which we use a standard calibration of the Taylor rule based on the existing literature (Fernández Villaverde, 2012; Christiano, Eichembaum and Rebelo, 2011, among many others). This kind of monetary policy is usually classified as active, following Leeper's definition, as the nominal interest rate reacts strongly to deviations of inflation from its steady state. In turn, the parameters of the fiscal rule are set to $d_g = -0.01$ and $d_y = 0$, meaning that fiscal policy is passive. We will refer to this case as Scenario 1 or Spanish scenario. The second scenario considers a passive monetary policy, according to Leeper, to represent a monetary policy more similar to the one implemented in the U.S. We set $d_g = -0.0001$ and $d_y = -0.01$, so that fiscal policy is active. We will call this Scenario 2 or U.S. scenario.

7.1 Alternative scenarios without macroprudential policy

Figure 1 shows the response of the economy to a 1% standard deviation increase in the credit risk shock, $\sigma_{\omega,t}$ in the absence of macroprudential policies. The figure displays the two policy scenarios just described: Spanish scenario/Scenario 1 (dashed) and US scenario/Scenario 2 (solid).

When the economy is hit by an increase in credit risk in the private sector, the probability of default of borrowers rises. Lenders will toughen the terms of the contract by increasing the state-contingent threshold value of the idiosyncratic shock (not shown in the figures) and the interest rate paid on loans. This generates a decrease in total private debt and therefore, a decrease in private investment. As a consequence, GDP falls on impact, in line with Christiano et al. (2010) and Gomes and Seoane (2017).

The fall of private loans also leads to a consequent decrease in the price of capital (Tobin's q). The firm's networth is directly related to the Tobin's q, as the latter establishes the value of the assets of the firm. Therefore, even if both private loans and net worth go down, the shock generates a shift from the share of capital investments that are financed by the entrepreneur's own resources to the share of capital investments

[19] We leave aside the tax rule to isolate the effects of a fiscal policy that uses a government spending rule exclusively.

financed by external funds. As a consequence the external finance premium goes up. The credit risk shock generates a decrease in labor that implies a fall in output below its steady state for the first ten periods approximately. As taxes in our model are distortionary, they also decrease following the same path as output. Consequently, public debt goes up in line with the decrease in public revenues. Summing up, lower private debt depresses investment and output. Tax collection falls, pushing upwards public debt. Fiscal consolidation to control public leverage implies further adjustments in output.

The combination of monetary and fiscal policy translates into slightly different paths for the policy-related variables, with no significant change in the rest of aggregates. In particular, monetary and fiscal policies do not seem to substantially affect the financial sector in any of the scenarios considered, except for the effect of the debt deflation channel. The monetary and fiscal policy mix is practically irrelevant for the behavior of private debt. Active monetary policy cancels the effects of inflation in public debt and passive fiscal policy does not stabilize the latter completely. Passive monetary policy allows inflation to counteract only slightly the rise of public leverage caused by an active fiscal policy.

In both scenarios, the private-public debt mechanism is at work. Public and private debt move in opposite directions. Notice that, the mechanism operates no matter the combination of fiscal and monetary policy that is in place, as Figure 1 shows. In the absence of an explicit response to financial conditions, none of the monetary-fiscal policy mixes considered here is able to offset the amplifying effects of financial frictions in the event of a credit shock.

7.2 Alternative scenarios with macroprudential policy

We next analyze the previous scenarios when macroprudential policy is included in the model. As mentioned in Section 4, we consider macroprudential instruments that aim at stabilizing credit market conditions and do not address GDP directly. We set the elasticity of η_t to credit market conditions, γ_η, equal to 1.75, no matter what kind of macroprudential policy is in place, to ensure that our results are comparable. This means that the macroprudential instrument will increase in 1.75% for every 1% increase in the credit market conditions considered, and vice versa.

7.2.1 Dynamics to credit risk shock

Firstly, we introduce a macroprudential tool that targets nominal credit growth, as in equation (38). In a second step, a macroprudential instrument that targets the credit-to-GDP ratio is considered, that is, equation (39). These two macroprudential cases are represented in the figures by a dashed line and a dotted line, respectively. The results obtained are shown in Figures 2 and 4 and explained below. Figures 3 and 5 show the evolution of the credit market conditions under the different kinds of macroprudential regimes for each of the monetary-fiscal scenarios taken into account.

The introduction of macroprudential instruments has clear effects on the financial sector. Private debt is almost completely stabilized when the credit-to-GDP ratio is the objective. This is not the case when authorities focus on the growth rate of nominal debt although its volatility is reduced with respect to the no macroprudential case. In both cases, macroprudential policies respond to the downturn by easing credit conditions. Thus, putting less pressure on investment in response to the shock. This is passed on to output, alleviating public revenues, and allowing for a timid fiscal expansion without incurring in too much public debt.

Notice that, in both scenarios, targeting nominal private debt growth isolates net worth and Tobin's q from the negative effects of the shock. This contributes to output stabilization. Targeting nominal credit growth also increases loanable funds after a credit risk shock, but less than under the credit-to-GDP ratio. The effects of the shocks are smoothed but not enough to generate enough taxes as to feedback into more output. As a consequence, the response of the economy is smoothed attaining more overall stability.

The simulations show that the two macroprudential policies considered here can break the private-public mechanism found in the data. However, each macroprudential tool has different implications in terms of economic stability. When the credit-to-GDP ratio is the target, the positive correlation between private and public debt comes at the cost of more output volatility. The opposite is true under a macroprudential policy regime that tracks nominal credit growth. One key variable in this mechanism is the introduction of distortionary taxes. Procyclical tax collection can amplify the effects of macroprudential policies by allowing for a fiscal expansion without excessive public debt.

7.2.2 Volatility and correlations

Table 3 confirms the results described above. It reports the standard deviations of output, inflation, private and public debt under the scenarios considered. The macroprudential instrument that stabilizes output the most is the one that reacts to the nominal credit growth. Targeting credit-to-GDP attains the highest private debt stabilization. However, as mentioned above, this is at the cost of higher output volatility.

Finally, we look at the correlation between the private debt-to GDP ratio and the public debt-to-GDP ratio to evaluate if the private-public debt mechanism can be offset in any of the scenarios considered (see Table 4). We focus on these ratios to analyze a measure that is similar to the data that was collected in the empirical analysis of Section 1. We find that in all the cases in which macroprudential policy is introduced in the model, the negative correlation between both variables that was present in the baseline model disappears. Moreover, the macroprudential tool that stabilizes nominal credit growth is the one that offsets this channel the most, mainly by smoothing the path of public debt, as the correlation goes from negative to around 0.91. Targeting the credit-to-GDP ratio offsets the private-public debt channel mainly by smoothing the path of private debt. This translates into positive although lower correlations than the ones just described. These results reflect the debate on the choice of the macroprudential tool, given the different results obtained in this model in terms of economic stability.

7.2.3 Robustness analysis

To shed some light on this point, we next investigate the robustness of these results to the parameterization of γ_η. We follow standard procedure in the literature and compute policy frontiers in terms of the volatility of output and that of the macroprudential policy target for a grid of values for $\gamma_\eta = [0, 2]$. The results are displayed in Figure 6. In the figure, the first row corresponds to nominal growth of private debt as the policy objective, and the second row refers to the credit-to-GDP ratio. The colors in the plots become lighter the larger is γ_η. The triangle highlights the combination that corresponds to the current benchmark calibration of the model.

The trade-off between output and instrument stabilization is clear in the four cases considered, but depends on the value of γ_η. When credit-to-GDP is the target, macroprudential policy needs not be so aggressive to stabilize output: the model just needs $\gamma_\eta \geq 0.05$, versus $\gamma_\eta \geq 0.79$ for the nominal credit growth target. However, the target becomes more volatile (the vertical axis in the second row of the graph is around twice that in the first row). There are some values for which there is no trade-off between output and target stabilization. In both cases, higher values of γ_η attain lower output volatility, pointing in favor of aggressive macroprudential policies over the cycle. It is worth mentioning that for the nominal credit growth target under the passive monetary policy/active fiscal policy combination, there are some discontinuities around $\gamma_\eta = 1$ that lead to excessively large σ_y.[20]

Finally, we compute the loss attained in terms of volatility when the economy is affected by credit risk shocks. We assume a macroprudential authority who cares about output volatility and the variance of the policy instrument as follows:

$$\mathcal{L} = (1 - \phi_y)\sigma_\Psi^2 + \phi_y \sigma_y^2, \tag{44}$$

where $\phi_y \in [0, 1]$ represents the relative weights of σ_y and σ_Ψ in the objective function;[21] and where Ψ_t is given either by equation (38) or equation (39). We calculate the loss under the alternative policy-mix scenarios considered and for a range of parameters relating the relative weights of the objectives in the loss function. The results appear in Table 5.

The table displays absolute values for L in the cases of no macroprudential policy, when macroprudential policy targets nominal credit growth, and finally, when the objective is the ratio of credit to GDP. To assess the change in volatlity of using macroprudential policies, we compute the percentage deviation of the loss under each policy target under the benchmark calibration with respect to the no-macrorpudential policy case.

We observe that active monetary/passive fiscal policies reduce overall loss when nominal credit growth is the target. The same is true for the credit-to-GDP ratio as long as the macroprudential authority also cares about target stabilization. The passive monetary/active fiscal combinations work in favor of the credit-to-GDP ratio, again for $\phi_y < 1$. Notice that when monetary policy is passive and fiscal policy is active, the use of either target of macroprudential policy increases overall volatility. The reason is the weak reaction of interest rates that limit inflation stabilization.

The objective of this exercise was to understand the volatility implications of alternative setups, not to derive the optimal policy. This is, however, a relevant analysis which is left for future research.

8 Conclusion

During the period between 2007 and 2017 some countries, such as the US or Spain, that implemented different and even opposite fiscal policies, show a negative relation between their own levels of private and public debt. Based on these empirical facts, this paper

[20]These extreme points have been excluded from the current graph.

[21]Equation (44) represents a loss in the sense that less volatility is preferred to more.

shows that macroprudential policy is a complementary tool to the monetary-fiscal policy mix when the objective is to stabilize private and public debt at the same time.

We build a model with financial frictions, in which fiscal and monetary policies interact in response to a credit risk shock that brings an economic recession. As a consequence, a private-public debt mechanism arises by which private and public debt move in opposite directions, so the traditional macroeconomic policies cannot stabilize both variables at the same time. The intuition for this is the fact that when private debt decreases after a positive credit shock, investment also goes down reducing output. As taxes in our model are distortionary, the fall of GDP implies a reduction in public revenues and the consequent rise of public debt. We find that the standard mix of monetary and fiscal policies is not enough to stabilize both variables at the same time.

We then introduce macroprudential policy responding either to the nominal private credit growth or to changes in the private credit-to-GDP ratio. The analysis of these macroprudential tools is performed for two different scenarios that combine active/passive fiscal and monetary policies. The results show that macroprudential policy, no matter how it is designed, affects the lending-deposit spread in a way that offsets, at least partially, the private-public debt channel in the two scenarios considered, stabilizing private debt more than when there is no macroprudential policy in place. Nevertheless, macroprudential authorities need to consider both the interaction of macroprudential tools with the monetary and fiscal policies in place and the correct credit market variables that macroprudential policy should address, depending on whether they want to attain economic stability or more financial stability. A macroprudential tool that reacts to the nominal credit growth is the one that best offsets the private-public debt channel and it does it by stabilizing public debt. This macroprudential tool is the one that best stabilizes the business cycle by decreasing output volatility, specially when it is combined with a passive monetary policy and an active fiscal policy (Scenario 2). However, a macroprudential tool that targets the credit-to GDP ratio offsets the mechanism by stabilizing private debt. This is the macroprudential tool that best stabilizes private debt, specially under an active monetary policy and a passive fiscal policy (Scenario 1).

This calls to question to what extent Scenario 1 applies to Spain. In Scenario 1 monetary policy reacts to domestic conditions, but actually, the European Central Bank does not react to individual country's conditions. We leave for further research the replica of this analysis in an economy where monetary policy is constrained by, for instance, the belonging to a currency area. Malmierca (2019) performs this analysis adapting the model to the EMU countries, in what refers to monetary policy.

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Appendix A

Data

Data for Spain and the US cover the period 1960-2017 for the main series of interest. Data in Table 1 that cover the period 1960-2017 are: real government consolidated gross debt-to-real GDP ratio, D, real credit to the private non-financial sector-to-real GDP ratio, B, real GDP, Y, and real government final consumption expenditure, G. Real GDP and the GDP deflator were collected from the European Commission's AMECO Database. Real public debt is the deflated series of the nominal general government consolidated gross debt obtained from AMECO for Spain, and of the nominal total federal debt from the Federal Reserve of St. Louis' FRED Database for the US. Data on real private debt was generated by deflating the nominal series available at the Bank of International Settlements (BIS) on credit, from all sectors of the economy, to the private non-financial sector (non financial corporations, households and non-profit institutions serving households), adjusted for breaks. We also use OECD data on real government final consumption expenditure.

For the comparison of the data we detrend both the real GDP and the real public consumption applying the Hodrick Prescott filter. To evaluate real private and public debt we use their ratio over GDP.

Appendix B (Follows closely Bernanke et al. (1999) and Fernández-Villaverde (2010))

B.1 Costly state verification problem

In this section we solve the contract by considering jointly the problems of the entrepreneur and the financial intermediary.

The standard debt contract specifies the repayment to the financial intermediary that depends on the return that the entrepreneur gets from its investment project. This household-financial intermediary-entrepreneur relationship represents the financial sector of the economy in our model. The financial sector plays an important role in our analysis due to the fact that there is asymmetric information, that is, entrepreneurs do not always tell the truth about their realized return and therefore financial intermediaries should monitor entrepreneurs by paying an auditing cost in the event of default, μ. This implies that there is a moral hazard problem due to asymmetric information that can only be avoided through a costly state verification (CSV). The CSV is the reason why entrepreneurs will always find more expensive to finance their investments with external funds than with internal funds. We model this CSV following the one developed in Bernanke, Gertler and Gilchrist (1999) and that was first analyzed by Townsend (1979).

At the moment of the debt contract agreement there is aggregate uncertainty because R^k_{t+1} is not known yet. Once the entrepreneur has decided on the amount of capital he wants to purchase, that is, his expenditures for period t, $q_t k_t$, and therefore the amount of external funds that he needs, the entrepreneur and the financial intermediary agree to sign a one period contract given the ex-ante values of $q_t k_t$ and $\frac{b_t}{p_t}$. The contract will establish a state-contingent non-default repayment R^l_{t+1} (dependent on the ex-post realization of R^k_{t+1}) that the entrepreneur promises to pay to the financial intermediary in case he succeeds in his investment project, that is, if he obtains enough return to meet its payment obligations with the financial intermediary. Otherwise the entrepreneur will default. If the entrepreneur defaults, it gets nothing because the financial intermediary pays the auditing cost to monitor what the entrepreneur reports as his revenue. Then the financial intermediary takes the remaining fraction $(1-\mu)$ of the entrepreneur's return because the rest of that return (fraction μ) is lost in bankruptcy procedures. Hence, the CSV problem is designed in such a way that ensures that if the entrepreneur has generated enough revenue to pay his obligations he has an incentive to do so and to report truthfully. This is what Freixas and Rochet (2008) call the revelation mechanism.

The value of the idiosyncratic shock, ω_{t+1}, is only known by the borrower, what causes the information asymmetries that generate a costly state verification. The moral hazard problem that arises from the asymmetric information implies that a standard debt contract with CSV is necessary. This contract establishes the main terms of the borrowing relation between the financial intermediary and the entrepreneur. Then, as a result of the moral hazard problem, the standard debt contract also specifies a state-contingent threshold value of the idiosyncratic shock ϖ_{t+1} (dependent on the ex-post realization of R^k_{t+1}), below which the entrepreneur defaults because according to the following equation he cannot pay back his debt.

$$R^l_{t+1} b_t = \varpi_{t+1} R^k_{t+1} p_t q_t k_t. \tag{B.1.1}$$

Summarizing, after the idiosyncratic shock is realized there are two possible scenarios:

- if $\omega_{t+1} > \varpi_{t+1}$ the financial intermediary will get $R^l_{t+1} b_t$ and the entrepreneur will keep the difference between his revenue and the interest payment on the loan,

$\omega_{t+1} R^k_{t+1} p_t q_t k_t - R^l_{t+1} b_t$;

- if $\omega_{t+1} < \varpi_{t+1}$ the entrepreneur defaults and he gets nothing while the financial intermediary gets $(1-\mu)\,\omega_{t+1} R^k_{t+1} p_t q_t k_t$ and $\mu \omega_{t+1} R^k_{t+1} p_t q_t k_t$ is a monitoring cost.

The debt contract also establishes that the return that the financial intermediary gets from the entrepreneur, no matter he pays or defaults, gives zero profits to the former. Therefore, the zero profit condition of the financial intermediary is the following:

$$\left[1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] R^l_{t+1} b_t + (1-\mu) \int_0^{\varpi_{t+1}} \omega dF\left(\omega, \sigma_{\omega,t}\right) R^k_{t+1} p_t q_t k_t = R_t a_t. \tag{B.1.2}$$

The previous expression shows that the potential revenues that the financial intermediary will obtain from its relation with the borrower must be equal to the cost of funds that the financial intermediary has to assume in his relation with the household.

If we consider equation $R^l_{t+1} b_t = \varpi_{t+1} R^k_{t+1} p_t q_t k_t$ we get:

$$\frac{R^k_{t+1}}{R_t} \left[\varpi_{t+1} \left[1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] + (1-\mu) \int_0^{\varpi_{t+1}} \omega dF\left(\omega, \sigma_{\omega,t}\right) \right] q_t k_t = \frac{a_t}{p_t}. \tag{B.1.3}$$

Because of the market clearing conditions that will be explained later, the total amount of deposits in the economy should equal the total amount of loans in the economy:

$$a_t = b_t, \tag{B.1.4}$$

so we can rewrite the previous equation as:

$$\frac{R^k_{t+1}}{R_t} \left[\varpi_{t+1} \left[1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] + (1-\mu) \int_0^{\varpi_{t+1}} \omega dF\left(\omega, \sigma_{\omega,t}\right) \right] q_t k_t = \frac{b_t}{p_t}, \tag{B.1.5}$$

and taking into account the properties of the lognormal distribution, the zero profit condition of the financial intermediary can be written as:

$$\frac{R^k_{t+1}}{R_t} \left[\Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu G\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] q_t k_t = \frac{b_t}{p_t}, \tag{B.1.6}$$

where the probability of default is defined as:

$$G\left(\varpi_{t+1},\sigma_{\omega,t}\right)=\int_{0}^{\varpi_{t+1}}\omega dF\left(\omega,\sigma_{\omega,t}\right), \tag{B.1.7}$$

and where the share of entrepreneurial earnings accrued to the financial intermediary is:

$$\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)=\varpi_{t+1}\left[1-F\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]+G\left(\varpi_{t+1},\sigma_{\omega,t}\right). \tag{B.1.8}$$

Let us assume, for aggregation purposes, that all the entrepreneurs will have the same ratio of leverage over wealth, $\frac{\frac{b_t}{p_t}}{n_t}$, regardless their level of wealth. The problem of maximization of its expected net worth requires choosing both the ratio of leverage and the schedule for ϖ_{t+1}.

$$\max_{\frac{\frac{b_t}{p_t}}{n_t},\varpi_{t+1}}\frac{R_{t+1}^{k}}{R_t}\left[1-\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]\left(1+\frac{\frac{b_t}{p_t}}{n_t}\right), \tag{B.1.9}$$

subject to the zero profit condition of the financial intermediary,

$$\left[\frac{R_{t+1}^{k}}{R_t}\left[\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]\left(1+\frac{\frac{b_t}{p_t}}{n_t}\right)-\frac{\frac{b_t}{p_t}}{n_t}\right]. \tag{B.1.10}$$

After maximizing the previous expression we get two first order conditions with ξ_t as the Lagrangian coefficient:

$$\begin{aligned}&E_t\frac{R_{t+1}^{k}}{R_t}\left[1-\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]\\+\xi_t&\left\{\frac{R_{t+1}^{k}}{R_t}\left[\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]-1\right\}=0,\end{aligned} \tag{B.1.11}$$

and

$$-\Gamma_{\omega}\left(\varpi_{t+1},\sigma_{\omega,t}\right)+\xi_t\left[\Gamma_{\omega}\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]=0. \tag{B.1.12}$$

From this last first other condition we can write the Lagrangian as:

$$\begin{aligned}\xi_t=&\frac{\Gamma_{\omega}\left(\varpi_{t+1},\sigma_{\omega,t}\right)}{\Gamma_{\omega}\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)}=\\&\frac{1-F\left(\varpi_{t+1},\sigma_{\omega,t}\right)}{1-F\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu\varpi_{t+1}F_{\omega}\left(\varpi_{t+1},\sigma_{\omega,t}\right)},\end{aligned} \tag{B.1.13}$$

and then rewriting we get:

$$E_t \frac{R_{t+1}^k}{R_t} \left[1 - \Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] = E_t \left[\frac{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)}{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu \varpi_{t+1} F_\omega \left(\varpi_{t+1}, \sigma_{\omega,t}\right)} \right] \left\{ 1 - \frac{R_{t+1}^k}{R_t} \left[\Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu G\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] \right\}, \tag{B.1.14}$$

what combined with the zero profit condition of the financial intermediary gives:

$$E_t \frac{R_{t+1}^k}{R_t} \left[1 - \Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] = E_t \left[\frac{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)}{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu \varpi_{t+1} F_\omega \left(\varpi_{t+1}, \sigma_{\omega,t}\right)} \right] \frac{n_t}{q_t k_t}, \tag{B.1.15}$$

also written as:

$$q_t k_t = \left[\frac{\xi_t}{E_t \frac{R_{t+1}^k}{R_t} \left[1 - \Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right]} \right] n_t. \tag{B.1.16}$$

where $q_t k_t$ are purchases of capital, as explained before and where $\frac{R_{t+1}^k}{R_t}$ is the external finance premium, inversely related to the net wealth of the entrepreneur. This implies that, everything else equal, a rise in the external finance premium, $efp = \frac{R_{t+1}^k}{R_t}$ that initially reduces the expected probability of default, generates a decrease in net worth relative to external funds and therefore ends up increasing the expected probability of default.

We also assume that at the end of the period a fraction γ^e of entrepreneurs survives while the rest die[22]. Then the net wealth of the dead entrepreneurs, $(1 - \gamma^e)\, n_t$, is paid back to households and they transfer w^e to incoming entrepreneurs, that is the initial real net wealth with which the new entrepreneurs that substitute the dead ones enter the model. For simplicity, this initial real net wealth is received by all the entrepreneurs. Therefore, households receive from entrepreneurs the net transfer defined before:

$$tre_t = (1 - \gamma^e)\, n_t - w^e; \tag{B.1.17}$$

[22] Capital demand and capital return by entrepreneurs depend on the evolution of their net worth. And at the same time, entrepreneurs' net worth (equity) depends on their earnings net of interest payments to financial intermediaries. Therefore it is necessary to assume that entrepreneurs have some initial networth, tre_t, in order to begin operating.

The average net wealth (equal to the wealth of the entrepreneur since all the entrepreneurs get the same leverage ratio) is:

$$n_t = \gamma^e \frac{1}{\Pi_t} \left\{ \left[1 - \mu G\left(\varpi_t, \sigma_{\omega,t-1}\right)\right] R_t^k q_{t-1} k_{t-1} - R_{t-1} \frac{b_{t-1}}{p_{t-1}} \right\} + w^e. \tag{B.1.18}$$

B.2 Contract between financial intermediary and entrepreneur

Our model includes a productivity shock ω_{t+1} that is lognormally distributed with a cumulative distribution function represented by $F(\omega, \sigma_{\omega,t})$ and $\mu_{\omega,t}$ as the average and $\sigma_{\omega,t}$ as the standard deviation of the distribution where $E_t\omega_{t+1} = 1$. From the properties of the lognormal distribution we have:

$$E_t\omega_{t+1} = e^{\mu_{\omega,t} + \frac{1}{2}\sigma^2_{\omega,t}} \Rightarrow e^{\mu_{\omega,t} + \frac{1}{2}\sigma^2_{\omega,t}} = 1 \Rightarrow \mu_{\omega,t} + \frac{1}{2}\sigma^2_{\omega,t} = 0 \Rightarrow \mu_{\omega,t} = -\frac{1}{2}\sigma^2_{\omega,t}.$$

In the computations to obtain the loglinearized version of the model we use the following equations that are also derived from the properties of the lognormal distribution:

$$\Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right) = \varpi_{t+1}\left(1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right) + G\left(\varpi_{t+1}, \sigma_{\omega,t}\right),$$

$$\Gamma_\omega\left(\varpi_{t+1}, \sigma_{\omega,t}\right) = 1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right),$$

$$G\left(\varpi_{t+1}, \sigma_{\omega,t}\right) = 1 - \phi\left(\frac{\frac{1}{2}\sigma^2_{\omega,t} - \log\varpi_{t+1}}{\sigma_{\omega,t}}\right),$$

and

$$G_\omega\left(\varpi_{t+1}, \sigma_{\omega,t}\right) = \varpi_{t+1} F_\omega\left(\varpi_{t+1}, \sigma_{\omega,t}\right).$$

Appendix C

Entrepreneur's problem with macroprudential policy

We solve again the problem of the entrepreneur introducing the macroprudential tool in the zero profit condition of the financial intermediary. Therefore, we have that:

$$\frac{R_{t+1}^{k}}{R_t}\left[\varpi_{t+1}\left[1-F\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]+(1-\mu)\int_0^{\varpi_{t+1}}\omega dF\left(\omega,\sigma_{\omega,t}\right)\right]q_t k_t=\eta_t\frac{b_t}{p_t}, \tag{C.1}$$

and taking into account the properties of the lognormal distribution, we now write the zero profit condition of the financial intermediary as:

$$\frac{R_{t+1}^{k}}{R_t}\left[\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]q_t k_t=\eta_t\frac{b_t}{p_t}, \tag{C.2}$$

The problem of maximization of the entrepreneur's expected net worth requires choosing both the ratio of leverage and the schedule for ϖ_{t+1}

$$\max_{\frac{\frac{b_t}{p_t}}{n_t},\varpi_{t+1}}\frac{R_{t+1}^{k}}{R_t}\left[1-\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]\left(1+\frac{\frac{b_t}{p_t}}{n_t}\right), \tag{C.3}$$

subject to the zero profit condition of the financial intermediary,

$$\left[\frac{R_{t+1}^{k}}{R_t}\left[\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]\left(1+\frac{\frac{b_t}{p_t}}{n_t}\right)-\eta_t\frac{\frac{b_t}{p_t}}{n_t}\right]. \tag{C.4}$$

After maximizing the previous expression we get two first order conditions with ξ_t as the Lagrangian coefficient:

$$E_t\frac{R_{t+1}^{k}}{R_t}\left[1-\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]+\xi_t\left\{\frac{R_{t+1}^{k}}{R_t}\left[\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]-\eta_t\right\}=0, \tag{C.5}$$

and

$$-\Gamma_\omega\left(\varpi_{t+1},\sigma_{\omega,t}\right)+\xi_t\left[\Gamma_\omega\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]=0. \tag{C.6}$$

From this last first other condition we can write the Lagrangian as:

$$\xi_t=\frac{\Gamma_\omega\left(\varpi_{t+1},\sigma_{\omega,t}\right)}{\Gamma_\omega\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)}=\frac{1-F\left(\varpi_{t+1},\sigma_{\omega,t}\right)}{1-F\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu\varpi_{t+1}F_\omega\left(\varpi_{t+1},\sigma_{\omega,t}\right)}, \tag{C.7}$$

and then rewriting we get:

$$E_t \frac{R^k_{t+1}}{R_t} \left[1 - \Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] =$$

$$E_t \left[\frac{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)}{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu \varpi_{t+1} F_\omega\left(\varpi_{t+1}, \sigma_{\omega,t}\right)} \right]$$

$$\left\{ \eta_t - \frac{R^k_{t+1}}{R_t} \left[\Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu G\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] \right\}, \quad \text{(C.8)}$$

what combined with the zero profit condition of the financial intermediary gives:

$$E_t \frac{R^k_{t+1}}{R_t} \left[1 - \Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] =$$

$$E_t \left[\frac{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)}{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu \varpi_{t+1} F_\omega\left(\varpi_{t+1}, \sigma_{\omega,t}\right)} \right] \eta_t \frac{n_t}{q_t k_t}, \quad \text{(C.9)}$$

also written as:

$$q_t k_t = \left[\frac{\xi_t \eta_t}{E_t \frac{R^k_{t+1}}{R_t} \left[1 - \Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right]} \right] n_t. \quad \text{(C.10)}$$

Finally, the average net wealth of the entrepreneur, taking into account the macroprudential instrument, becomes:

$$n_t = \gamma^e \frac{1}{\Pi_t} \left\{ \left[1 - \mu G\left(\varpi_t, \sigma_{\omega,t-1}\right)\right] R^k_t q_{t-1} k_{t-1} - R_{t-1} \frac{b_{t-1}}{p_{t-1}} \eta_t \right\} + w^e. \quad \text{(C.11)}$$

Appendix D

Equilibrium loglinearized equations of the model

The final loglinearized equations of the model without macroprudential policy can be found in Fernández Villaverde (2010). When macroprudential policy is introduced in the model the following equations differ from those of the baseline model and a new equation is included for the macroprudential tool. We assume two possible characterizations for the latter that depend on the definition of Ψ_t.

Equation for wealth evolution:

$$\widehat{n}_t = a_1\left(-\widehat{\Pi}_t\right) + a_2\left(\omega_c\widehat{\overline{\omega}}_t + \sigma_c\widehat{\sigma}_{\omega,t-1}\right) + a_3\left(\widehat{R}^k_t + \widehat{q}_{t-1} + \widehat{k}_{t-1}\right) + a_4\left(\widehat{R}_{t-1} + \widehat{\eta}_{t-1} + \widehat{\overline{b}}_{t-1}\right), \quad \text{(D.1)}$$

with,

$$a_1 = \frac{\gamma^e}{\Pi n}\left[\left(1-\mu G\left(\varpi\right)\right)R^k k - R\bar{b}\eta\right],$$

$$a_2 = -\frac{\gamma^e}{\Pi n}\mu G\left(\varpi\right)R^k k,$$

$$a_3 = \frac{\gamma^e}{\Pi n}\left(1-\mu G\left(\varpi\right)\right)R^k k,$$

and

$$a_4 = -\frac{\gamma^e}{\Pi n}R\bar{b}\eta.$$

Equation for entrepreneur 1:

$$E_t\widehat{R}^k_{t+1} - \widehat{R}_t + \omega_a E_t\widehat{\varpi}_{t+1} + \sigma_a\widehat{\sigma}_{\omega,t} = \widehat{n}_t - \widehat{q}_t - \widehat{k}_t + \widehat{\eta}_t. \quad \text{(D.2)}$$

Equation for entrepreneur 2:

$$\widehat{R}^k_t - \widehat{R}_{t-1} + \omega_b\widehat{\overline{\omega}}_t + \sigma_b\widehat{\sigma}_{\omega,t-1} = \widehat{\overline{b}}_{t-1} - \widehat{q}_{t-1} - \widehat{k}_{t-1} + \widehat{\eta}_t. \quad \text{(D.3)}$$

Equation for Macroprudential instrument that depends on nominal credit growth:

$$\widehat{\eta}_t = \gamma_\eta\left(\widehat{\overline{b}}_t - \widehat{\overline{b}}_{t-1} + \widehat{\Pi}_t\right). \quad \text{(D.4)}$$

Equation for Macroprudential instrument that depends on credit-to-GDP ratio:

$$\widehat{\eta}_t = \gamma_\eta\left(\widehat{\overline{b}}_t - \widehat{y}_t\right). \quad \text{(D.5)}$$

Notice that in the last two equations variable $\widehat{\Psi}_t$ is replaced by the variables that define it.

Tables

Table 2: Calibration of the parameters and steady states

Parameter	Description	Value	Source
β	Discount factor	0.999	Fernández-Villaverde (2010)
h	Consumption habits	0.5	Fernández-Villaverde (2010)
ϑ	Frisch elasticity of labor	0.5	Fernández-Villaverde (2010)
α	Capital share of the intermediate production function	0.33	Fernández-Villaverde (2012)
δ	Capital depreciation rate	0.023	Fernández-Villaverde (2012)
θ	Calvo pricing parameter	0.8	Fernández-Villaverde (2010)
ε	Elasticity of substitution across goods	8.577	Fernández-Villaverde (2012)
χ	Degree of indexation	0.6	Fernández-Villaverde (2010)
$pdef$	Annual probability of default	0.03	Bernanke et al. (1999)
μ	Bankruptcy costs	0.15	Fernández-Villaverde (2012)
γ^e	Survival rate of entrepreneurs	0.975	Fernández-Villaverde (2010)
τ_l	Steady state of labor income tax rate	0.24	Fernández-Villaverde (2010)
τ_r	Steady state of capital income tax rate	0.32	Fernández-Villaverde (2010)
Π	Target gross inflation	1.005	Fernández-Villaverde (2012)
l	Time devoted to work	1/3	Fernández-Villaverde (2010)
q	Tobin's q. Price of capital	1	Fernández-Villaverde (2010)
R^d	Steady state of interest rate on public debt	$\frac{\Pi}{\beta}$	Fernández-Villaverde (2010)
R	Steady state of interest rate on deposits	$\frac{R^d-1}{1-\tau_R}+1$	Fernández-Villaverde (2010)
$\frac{b}{k}$	Loan-to-capital ratio	1/3	Fernández-Villaverde (2010)
$\frac{g}{y}$	Government expenditure-to-GDP ratio	0.2	Gomes and Seoane (2017)
$\frac{d}{y}$	Public debt-to-GDP ratio	0.6	Gomes and Seoane (2017)
$S''[1]$	Capital adjustment costs	14.477	Fernández-Villaverde (2012)
γ_g	Persistence parameter of government spending shock	0.95	Fernández-Villaverde (2012)
σ_g	Volatility of government spending shock	0.007	Gomes and Seoane (2017)
ρ_z	Persistence of technology shock	0.95	Fernández-Villaverde (2012)
σ_z	Volatility of technology shock	0.007	Gomes and Seoane (2017)
ρ_σ	Persistence of credit risk shock	0.95	Fernández-Villaverde (2012)
η_σ	Volatility of credit risk shock	0.560	Gomes and Seoane (2017)
γ_R	Persistence of monetary policy shock	0.95	Fernández-Villaverde (2012)
σ_m	Volatility of monetary policy shock	0.003	Gomes and Seoane (2017)

Table 2: Calibration of the parameters and steady states

Parameter	**Description**	**Value**	**Source**
$\gamma_{\Pi}(1-\gamma_R)$	Response of intervention rate to changes in inflation	1.5 or 0.07	Scenario analysis
d_g	Response of government spending to changes in public debt	-0.01 or -0.0001	Scenario analysis
d_y	Response of government spending to changes in output	0 or -0.01	Scenario analysis
γ_η	Response of macroprudential tool to changes in credit market conditions	0 or 1.75	Own calibration

Table 3: Standard deviations under alternative policy mixes

Variable	No macroprudential tool	Credit-to-GDP ratio	Nominal credit growth
Scenario 1: active monetary - passive fiscal policies			
Output	0.0190	0.0335 (77%)	0.0152 (-20%)
Inflation	0.0010	0.0014 (39%)	0.0004 (-57%)
Public debt	0.0490	0.0641 (31%)	0.0252 (-49%)
Private debt	0.0722	0.0110 (-85%)	0.0457 (-37%)
Scenario 2: passive monetary - active fiscal policies			
Output	0.0234	0.0457 (95%)	0.0158 (-33%)
Inflation	0.0034	0.0096 (179%)	0.0009 (-75%)
Public debt	0.0558	0.1278 (129%)	0.0271 (-43%)
Private debt	0.0767	0.0142 (-81%)	0.0440 (-51%)

Note: These results are the standard deviations to a standard deviation credit risk shock with $\eta_\sigma = 0.560$. The numbers in brackets represent the percentage variation for each variable volatility with respect to its baseline scenario value.

Table 4: Correlation between public and private debt

	No macroprudential tool	Credit-to-GDP ratio	Nominal credit growth
Scenario 1	-0.6351	0.3447	0.9117
Scenario 2	-0.8251	0.8506	0.9147

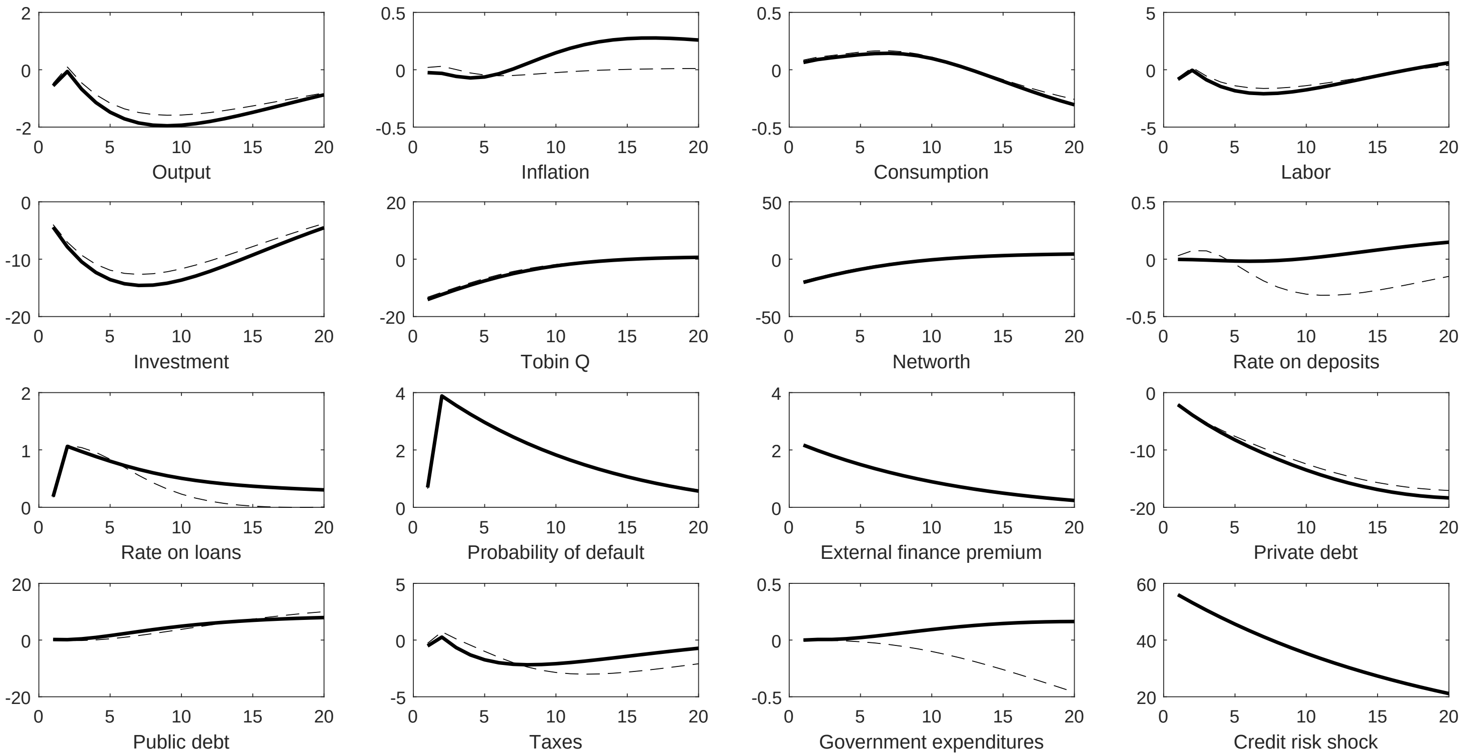
Table 5: Loss function for alternative scenarios and parameter values.

Policy scenario	γ_η	AM/PF $\phi_y = 0$	AM/PF $\phi_y = 0.5$	AM/PF $\phi_y = 1$	PM/AF $\phi_y = 0$	PM/AF $\phi_y = 0.5$	PM/AF $\phi_y = 1$
No macroprudential policy							
Nominal credit growth	0	0.0043	0.0185	0.0326	0.0035	0.0179	0.0323
Credit-to-GDP	0	0.0052	0.0028	0.0004	0.0059	0.0032	0.0005
Target: nominal credit growth	1.75	0.0016	0.0136	0.0255	0.0016	0.0246	0.0476
Change in overall volatility (%)		*-62.79%*	*-26.49%*	*-21.78%*	*-54.29%*	*37.43%*	*47.37%*
Target: credit-to-GDP ratio	1.75	0.0001	0.0006	0.0011	0.0002	0.0011	0.0021
Change in overall volatility (%)		*-98.08%*	*-78.57%*	*175%*	*-96.61%*	*-65.62%*	*320%*

Note: The changes in overall volatility are computed for each scenario with respect to the corresponding non-macroprudential case. In the table, AM/PF stands for active monetary/passive fiscal policies, and PM/AF refers to the case of passive monetary/active fiscal.

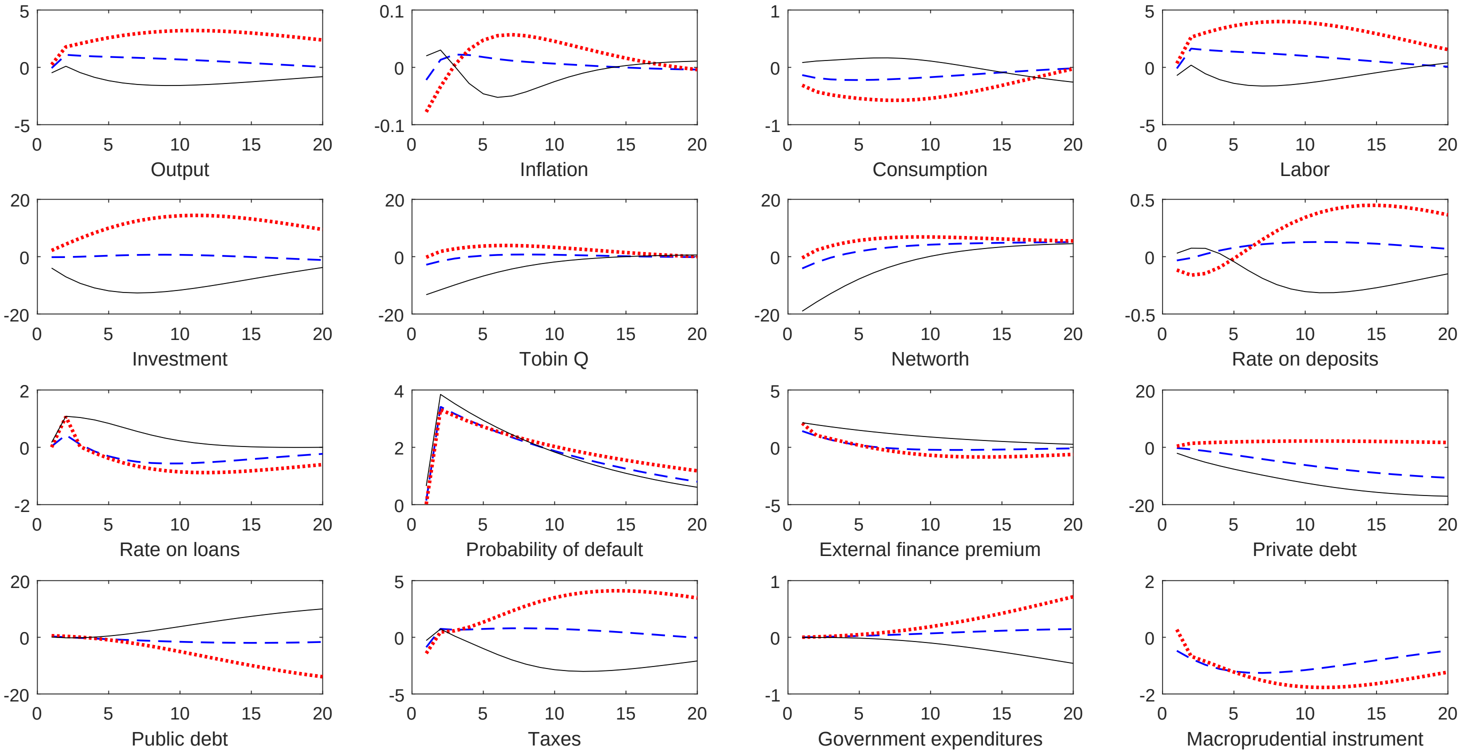
Figures

Figure 1: Impulse response functions to a 1 standard deviation rise in credit risk. Scenarios without macroprudential policy



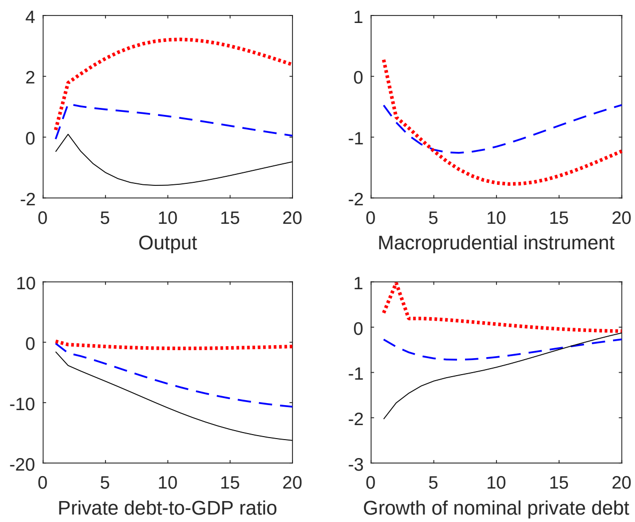
Note: The lines plotted in these graphs depict the IRFS for the cases without no macroprudential policy. The dashed line refers to Scenario 1 and the solid line refers to Scenario 2. Variables are expressed in percentage points of deviations from steady state.

Figure 2: IRFS to a 1 standard deviation rise in credit risk. Scenario 1.



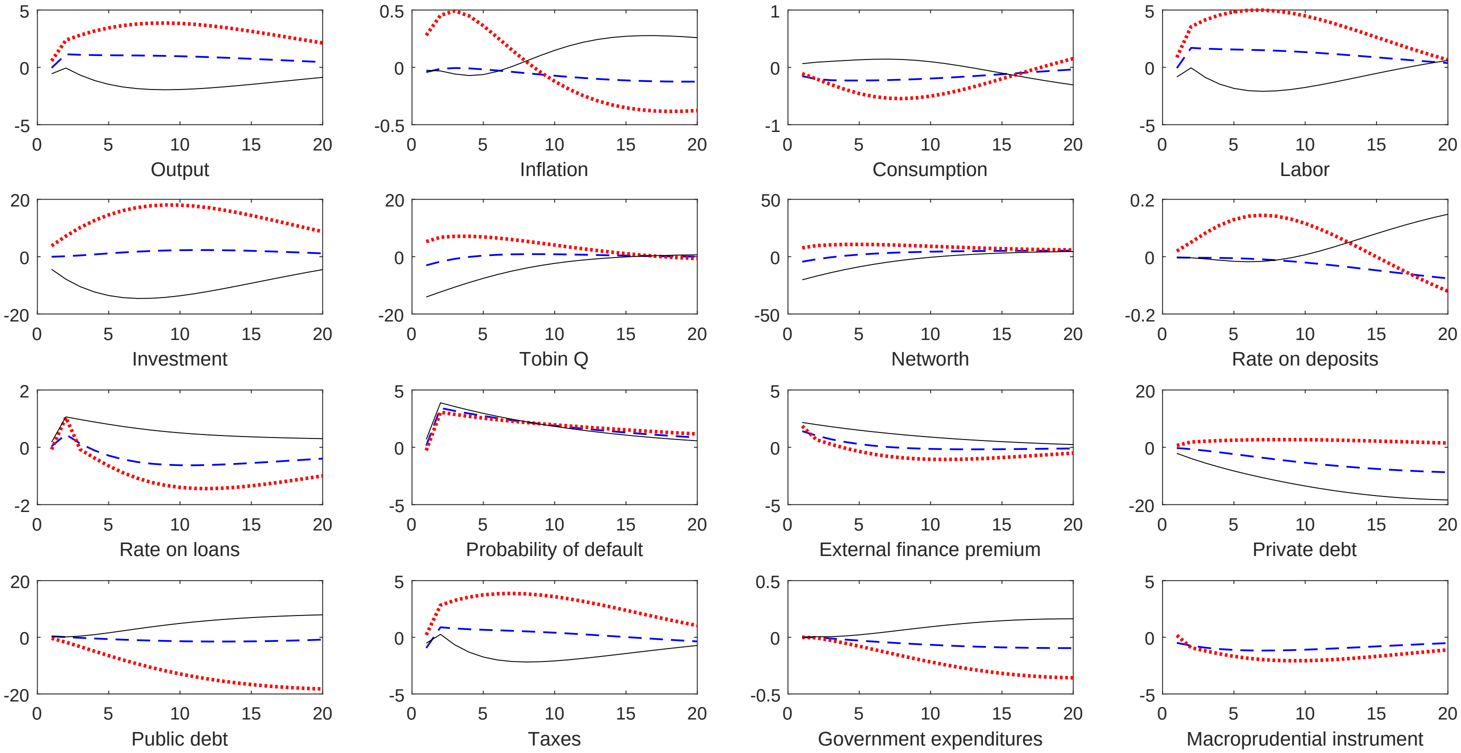
Note: The dashed line refers to to the case in which macroprudential policy targets the nominal credit growth, the dotted line to the case in which macroprudential policy targets the credit-to-GDP ratio and the solid line to the model without macroprudential policy. Variables are expressed in percentage points of deviations from steady state.

Figure 3: IRFS to a 1 standard deviation rise in credit risk. Credit market conditions in Scenario 1.



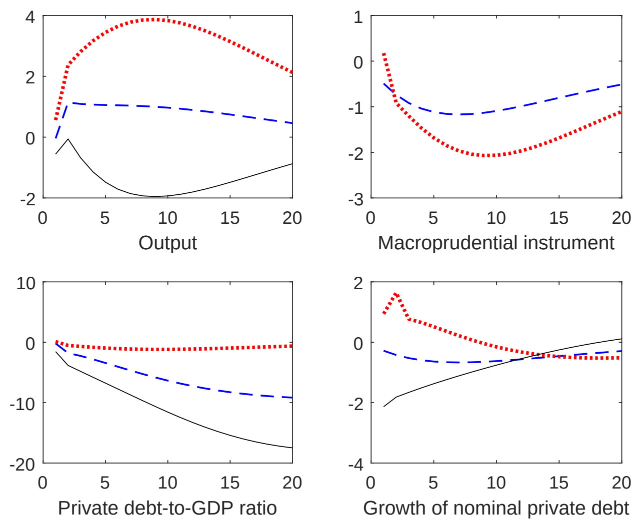
Note: The dashed line refers to to the case in which macroprudential policy targets the nominal credit growth, the dotted line to the case in which macroprudential policy targets the credit-to-GDP ratio and the solid line to the model without macroprudential policy. Variables are expressed in percentage points of deviations from steady state.

Figure 4: IRFS to a 1 standard deviation rise in credit risk. Scenario 2.



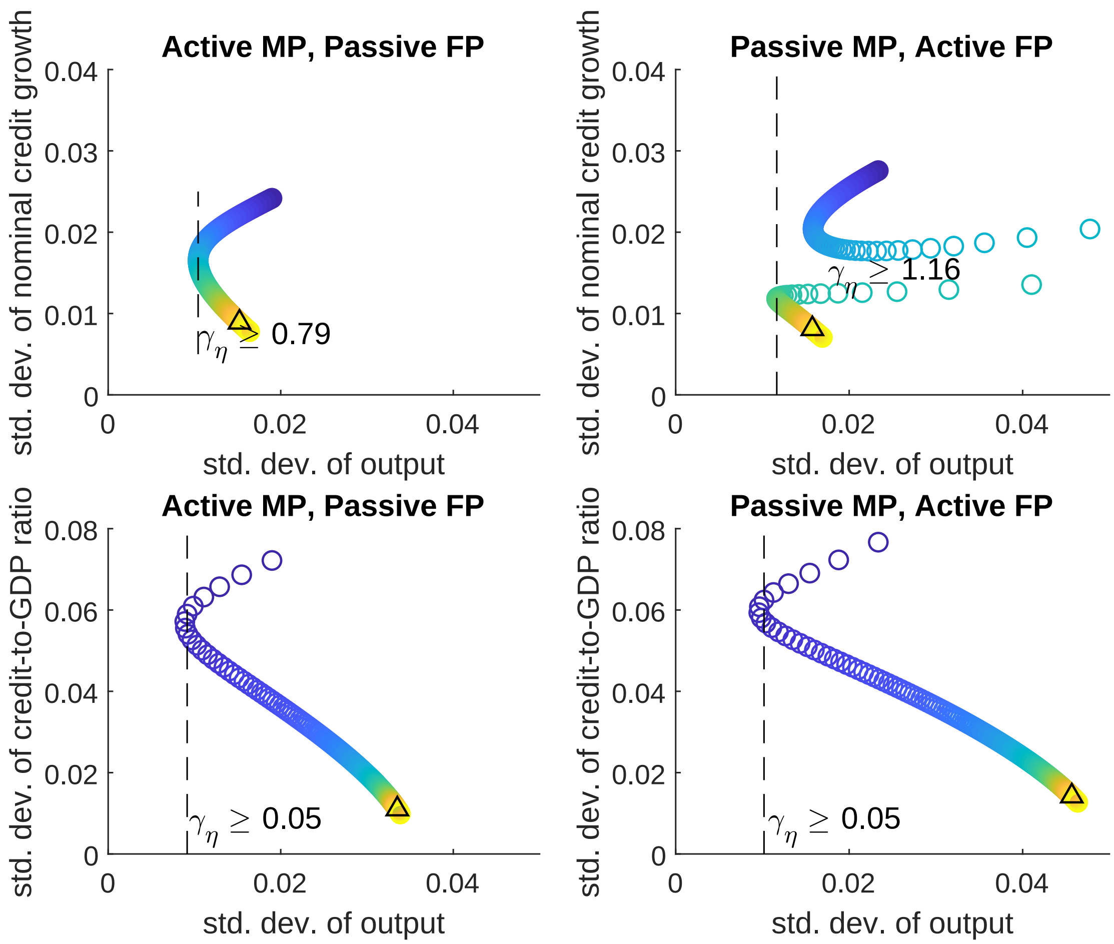
Note: The dashed line refers to to the case in which macroprudential policy targets the nominal credit growth, the dotted line to the case in which macroprudential policy targets the credit-to-GDP ratio and the solid line to the model without macroprudential policy. Variables are expressed in percentage points of deviations from steady state.

Figure 5: IRFS to a 1 standard deviation rise in credit risk. Credit market conditions in Scenario 2.



Note: The dashed line refers to to the case in which macroprudential policy targets the nominal credit growth, the dotted line to the case in which macroprudential policy targets the credit-to-GDP ratio and the solid line to the model without macroprudential policy. Variables are expressed in percentage points of deviations from steady state.

Figure 6: Robustness results to alternative γ_η.



Note: The first row corresponds to nominal growth of private debt as the policy objective, and the second row refers to the credit-to-GDP ratio. The colors in the plots become lighter the larger is γ_η. The triangle highlights the combination that corresponds to the current benchmark calibration of the model. The vertical line delimits relevant outcomes given the range of values for γ_η.