Stabilization and the policy mix in a monetary union *

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Abstract

The need for macroprudential policy to "lean against the wind" of credit cycles at the aim of financial and hence macroeconomic stability is a common belief. Which design of macroprudential policy might attain the greatest stability for the economy is still an open debate. This paper builds a two-country DSGE model for a monetary union and analyzes, through different macroprudential scenarios, the response of the main variables to an asymmetric credit risk shock. When national macroprudential policies are implemented, macroeconomic and financial stability is reached in both countries, mainly due to the cancellation of the *private-public debt channel*. When macroprudential policies are supranational, macroeconomic stability is higher in the country that suffers the shock while the other country is destabilized, mainly due to the *open economy channel*.

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1 Introduction

The stabilization differences in the EMU have called into question the ability of the traditional policies (monetary and fiscal) to stabilize by themselves a monetary union. There is an open debate about how authorities can complement the different national fiscal policies when monetary policy is centrally implemented and cannot address the particular needs of each country. Many authors claim a new instrument is needed to complement the traditional policies in a monetary union and to prevent, more than cure, the effects that a financial crisis may bring not only to the financial sector but also to the whole economy. The present paper contributes to this debate by analyzing alternative policy mixes that pursue financial and macroeconomic stabilization in a monetary union.

The novelty of this work is that it models a monetary union hit by asymmetric shocks where, given that monetary policy cannot be used by national authorities, fiscal and macroprudential policies interact. The model shows that a positive financial shock, that increases the credit risk of the private sector, can generate a *private-public debt channel* through which the economy is destabilized. I find that this channel represents a barrier to stabilize the financial sector and the whole economy and cannot be canceled by implementing only fiscal and monetary policies. The reason is that the positive financial shock causes a financial crisis that might impose the obligation for private deleveraging, destabilizing the economy. To counteract this deleveraging, on the one hand, the limits of fiscal policy may not allow to issue more public debt and, on the other hand, the central bank may not overuse non-conventional monetary policy. Thus an additional instrument is needed to offset the private-public debt channel and stabilize GDP and the financial sector, counteracting private deleveraging.

The private-public debt channel operates in the following way:[1] after a financial shock that reduces the level of private debt, investment goes down what is translated in a lower level of GDP. The decrease of GDP causes a drop in the collection of taxes and the consequent rise of public debt. This inverse relation between private and public debt amplifies the business cycle and slows down the recovery of GDP. When fiscal authorities undertake a fiscal consolidation through government spending to reduce the levels of public debt, they contribute to an even deeper fall of GDP. As taxes are proportional, public revenues also go down so the final outcome is a further rise of public debt. If, on the other hand, fiscal authorities expand public expenditures to restore GDP, they might provoke a new increase in public debt that might not be compensated by the increase

[1] See de Blas and Malmierca (2019).

in public revenues. Thus, as fiscal policy may not be enough to offset the private-public debt channel, I propose the introduction of an alternative policy that does alter it: macroprudential policy. In my open economy model, the spillover effects of a financial shock, that increases credit risk and is originated in the home country, transmit this channel beyond the national borders. Furthermore, due to the open economy dimension of the model, I show that macroprudential policy can sometimes stabilize the economy even if the channel is not offset. This is the case in which macroprudential policy addresses union-wide aggregates. The stabilization is then achieved through an *open economy channel*.

Thus, the present paper considers how authorities can stabilize their economies through the implementation of macroprudential policies. With that aim, I build a two-country DSGE model for a monetary union following the set up used in Quint and Rabanal (2014) with financial frictions, modeled as in Bernanke et al. (1999). I introduce a macroprudential tool to control the amount of loans that the banking system can lend to the private sector by targeting the growth of nominal credit. Unlike Quint and Rabanal (2014), my research compares a scenario in which macroprudential policy is implemented at the national level with a scenario in which it is implemented centrally (supranational implementation), analyzing how both macroprudential designs interact with national fiscal policies in a monetary union. In this model, fiscal policy, implemented at the national level, follows a government spending rule aimed at stabilizing public debt. Monetary policy, implemented at the union level, is set according to a standard Taylor rule. There are international financial markets that work as in Quint and Rabanal (2014): international financial intermediaries take the surplus of private funds from one country and supply those funds to the other country that has a shortage of funds. This implies that when macroprudential policy is implemented in one country, the effects of this measure are transferred through the international financial markets, generating spillovers to the other country.

The analysis focuses on the case of a positive financial shock, which increases the level of credit risk, and is originated in the country acting as net international borrower, that enters into a recession. The other country of the union is indirectly affected by the financial shock too. The international borrower of the model represents a periphery country, such as Spain, and the international lender a core country, such as Germany.[2] In this framework, I compare different macroprudential scenarios, given an active (inflation

[2] As Bordo (2014) states *"There has been a build-up of TARGET liabilities since 2007 by some central banks (notably Greece, Ireland, Portugal, and Spain, or the GIPS), and of TARGET assets by Germany and others"*.

targeting) monetary policy, as the one implemented in the EMU, and two national passive fiscal policies, in line with Leeper (1991).

The economic recovery achieved by the countries in this model depends, to a large extent, on the macroeconomic policies implemented by them and their neighbors in response to the shock that triggers the recession. In a monetary union, countries cannot use their own monetary policy, and fiscal policy is left alone to face the problem of economic instability. But fiscal policy cannot affect the financial variables by itself (see de Blas and Malmierca, 2019), and financial sector stabilization is crucial to smooth the business cycle after a financial shock. This is why macroprudential policy can play an important role.

The results show that, after a financial recession, macroprudential policy brings back both financial and macroeconomic stability. This is because it manages to change the response of the private sector variables to financial shocks and sometimes even breaks the private-public debt channel. It is worth mentioning that the introduction of fiscal instruments is crucial for this analysis as its interaction with the financial variables is the cause for the private-public debt channel. Considering that there is no common consensus on how the new toolkit should be designed, I shed light on the different stabilization effects of alternative macroprudential policy scenarios in a monetary union. Thus, I analyze a first case of national macroprudential policy that succeeds in offsetting the private-public debt channel and brings financial and economic stabilization for both countries. Then, I study a second scenario in which macroprudential policy is implemented at the union level and does not manage to break the channel in any of the countries. In this case, macroprudential policy brings the greatest economic stabilization to the country that suffers the shock while it destabilizes the other country.

The paper is organized as follows. Section 2 describes the empirical evidence that motivates the analysis of this paper. In Section 3, I review the most significant literature, closely related to this research. Section 4 includes the description of the open economy model and macroprudential policies. Section 5 contains the equilibrium and market clearing conditions of the model. Section 6 presents the calibration of the parameters. In Section 7, I analyze the effects of a credit shock in the main economic variables of a country belonging to a monetary union. In Section 8 I robust the results to alternative shocks. Section 9 concludes.

2 Empirical evidence

After the financial crisis of 2007, countries of the European Monetary Union (hereinafter EMU) followed very different recovery patterns in terms of restitution of their pre-crisis levels of GDP, inflation or unemployment.[3] For instance, by 2011, Germany had already reached its 2007 GDP level but Spain was still immersed in a national income fall (see Bozio et al., 2015). These GDP paths are illustrated in Figure 1. The economic recovery divergences experienced by countries belonging to the EMU, during the years that followed the Great Recession, motivates this research.

Moreover, I find that the private-public debt channel that amplifies the business cycle, operates in some countries EMU countries during the Great Recession. Table 1 shows that the private-public debt channel is present in Germany and Spain (see the correlation between private and public debt for each country (B-D)), for the subsample 2007-2017, a decade characterized by a financial crisis mainly originated by a financial shock.[4] By contrast, the channel is not present in any of the countries during the whole period 1960-2017. As explained in de Blas and Malmierca (2019), the change in the correlation between private and public debt, during the yeasrs of the Great Recession, may be the result of the private deleveraging process that the financial crisis imposed. However, the whole period 1960-2017, may have been affected in a higher proportion by other shocks which counteracted the effects of financial shocks so that the channel did not arise. The table also displays the correlation between public debt and output (D-Y) and government spending and output (G-Y), respectively, in Germany and Spain, for both the whole period 1960-2007 and the subsample 2007-2017. From the last column of the table, it is also possible to observe that, during the period of the Great Recession, the German GDP followed a more stable path than the Spanish one.

These two countries are affected by the same monetary policy but different government spending policies during the periods being analyzed. Column $\rho(G, Y)$ shows that during the sub-period 2007-2017, Germany used countercyclical government spending policies while Spain used them procyclically, and both resulted in negative correlations of private and public debt. Thus, in line with the findings in de Blas and Malmierca (2019), the cyclicality of government spending might not be the main cause for the negative correlation that Germany and Spain presented between private and public debt during

[3] Henceforth, I will refer to this restitution as the "economic recovery".

[4] In Table 1, private debt (B) does not include private debt held by households. Including households' private debt in the series implies that the correlation between B and D for the subperiod 2007-2017 is -0.556 in the case of Spain and -0.211 in the case of the Germany. These values also certify the negative correlation between these two variables after 2007.

Table 1: Contemporaneous correlation among main debt and output aggregates and standard deviation of GDP in Germany and Spain

Period 1960-2017				
	$\rho(B,D)$	$\rho(D,Y)$	$\rho(G,Y)$	$\sigma(GDP)$
Germany	0.309	-0.700	-0.307	0.025
Spain	0.327	-0.458	0.577	0.027
Sub-period 2007-2017				
	$\rho(B,D)$	$\rho(D,Y)$	$\rho(G,Y)$	$\sigma(GDP)$
Germany	-0.173	-0.481	-0.521	0.019
Spain	-0.511	-0.820	0.672	0.037

Note: B denotes real private debt-to-real GDP ratio; D is real public debt-to-real GDP ratio; Y represents real GDP, and G is real government consumption. Both real GDP, Y, and real public consumption, G, have been detrended using the Hodrick Prescott filter. To evaluate real private and public debt I use their ratio over GDP. Source: See Appendix A.

the subperiod 2007-2017. Therefore, as the private-public debt channel may not depend on the cyclicality of government spending, it seems that these kind of fiscal measures cannot eliminate the channel by themselves.

The negative correlation between public debt and GDP in both countries implies that when GDP goes down public debt increases, among other things, consequence of a fall in tax collection. With a countercyclical government spending that stabilizes output after a recession, public debt may rise because taxes do not compensate the increase in public deficit. With a pro-cyclical government spending that goes down with output, the reduction of government spending may result in a still lower level of GDP, a decrease in public revenues and therefore public debt may rise. This may explain why the channel operates in both Germany and Spain, even if they undertook opposite government spending strategies.

Finally, Table 1 also contains the GDP volatility for each country. The inverse relation of private and public debt in Spain during the sub-period 2007-2017 comes together with a more volatile cycle than for the whole period 1960-2017. However, in Germany, even if private and public debt are negatively correlated during the sub-period 2007-2017, GDP remains more stable than in the previous years. This is understandable, among other reasons, given that Germany implemented a series of structural reforms between years 2002 and 2007 to the labor market and strengthened public finances (Bozio et al., 2015).

Figure 2 shows that the level of private debt-to-GDP ratio in Spain at the beginning of the financial crisis was significantly large compared to Germany. When an economy enters into a recession the initial economic conditions are a key determinant in the posterior

speed of the economic recovery and stabilization (Bordo and Haubrich, 2012). Hence, macroprudential measures are important in stabilizing the economy, not only during busts but also during booms, to prevent initial economic conditions from accentuating the negative effects of a financial recession. In line with this, the IMF (2013) defines the aim of macroprudential policy as prevention rather than cure, so macroprudential policy has an important role ensuring that the initial levels of debt of an economy are not excessive. By shrinking private leverage in good times, macroprudential measures may help economies to maintain private leveraging stable. This way they will not enter into a financial crisis with such high levels of debt and economic stability will be more easily achieved. Moreover, as Bordo and Haubrich (2012) explain, the steeper the expansion, the deeper the posterior recession.

3 Related literature

This work contributes to the DSGE literature that studies the macroprudential policy stabilization effects in a monetary union. More concretely, the paper fits in the macroprudential literature that discusses whether these policies attain more stability when they are implemented at the national level or at the union level.

The model is an open economy version of the Fernández-Villaverde (2010a) and Gomes and Seoane (2018) new Keynesian model with financial frictions modelled as in Bernanke et al. (1999). Fernández-Villaverde (2010b) studies the effects of fiscal policy focusing on the use of distortionary taxes and a fiscal rule for government spending in the presence of financial frictions. He finds that government spending shocks are more powerful in stimulating output than tax shocks. Similarly, I analyze the effects of fiscal policy in a model with financial frictions but for an open economy within a monetary union that is hit by a financial shock. I also study the stabilization properties of the policy mix but including macroprudential policy to observe its interaction with traditional policies. I borrow the relevance of risk shocks as a key element in the propagation financial instability from Christiano et al. (2010). These authors find that the risk shock is responsible for a great part of the business cycle fluctuations both in the Euro Area and in the US. They argue that the recent economic crisis was mainly driven by a risk or financial credit shock. The latter motivates the introduction of this kind of shocks in the model and the use of macroprudential policy to fight against their destabilizing effects.

This paper studies a channel previously analyzed in de Blas and Malmierca (2019) by which the public and private sector are negatively correlated after a credit risk shock.

Corsetti et al. (2013) also analyze a *sovereign risk channel* through which higher sovereign default risk raises the financing costs of the private sector resulting in an adverse effect on economic activity. Unlike Corsetti et al. (2013), I study the inverse relation in the levels of sovereign and private debt. Their framework refers to the zero lower bound (ZLB) but they stress that their analysis could carry through other situations where monetary policy is constrained. Similarly, I analyze how the channel operates when monetary policy is constrained, because countries belong to a monetary union, and I consider an alternative instrument to offset this channel: macroprudential policy.

In this model, the private-public debt channel is a consequence of a connection between the financial sector and the public sector caused by fiscal instruments. The channel, that propagates the destabilizing effects of the shock from the financial sector to the broader economy, cannot be offset by using only fiscal policy. To understand why fiscal policy is unable to cancel the channel it is worth emphasizing that fiscal policy can be classified as active or passive, as defined by Leeper (1991) and applied to an extensive literature (Gomes and Seoane, 2018 or de Blas and Malmierca, 2019). Countercyclical fiscal policies aimed at boosting GDP during recessions are active. This kind of policy generates still higher levels of public debt and, depending on the size of the multiplier, may not manage to boost output and increase public revenues. The stabilization of output through active fiscal policies implies a trade-off: when the aforementioned channel works, the increase in public debt coincides with a decrease in private debt that constrains investment and can make output go down even deeper. On the other hand, when fiscal policy is passive, in Leeper's terminology, it targets public debt stabilization. In a recession, when the level of output has fallen substantially and the economy supports high levels of public debt, a fiscal consolidation will be implemented to reduce the latter. But this fiscal strategy could reduce the GDP level even more and thus the collection of taxes, increasing public debt. Therefore, the procyclical fiscal policy might not achieve its objective of public debt reduction.

Gomes and Seoane (2018) argue that different combinations of active/passive monetary and fiscal policies (based on Leeper (1991) definitions) are able to explain the different recovery paths across countries. They advocate that, after the Great Recession, the US experienced a faster economic recovery than the EMU due to the accelerator effects of financial frictions combined with an active fiscal regime. By contrast, the Euro Area was characterized by implementing a passive fiscal regime. I also analyze different economic recovery paths originated by the divergences in the policy mix, motivating my research in the evolution of two EMU countries after the Great Recession: Germany and Spain.

Moreover, this model differs from Gomes and Seoane's model in the use of proportional taxation, and in that I consider a public spending rule (instead of a lump-sum tax rule). Another important difference that determines my results is that I develop an open economy model while these authors analyze the policy mix in a closed economy.

Regarding the open economy literature, Galí and Monacelli (2005) model a continuum of small open economies to analyze the fiscal-monetary policy mix when monetary policy is set by a common central bank. They find that, under nominal rigidities, the lack of a national monetary policy requires that national fiscal policy assumes the stabilization role. I argue that, after a credit risk shock, stabilization, in terms of reduction of GDP and debt volatility, cannot be attained only by national fiscal policies. Therefore, what most differentiates my study from theirs is that I introduce macroprudential policy for the search of macroeconomic and financial stability in the monetary union.

I lay out a two-country model for a monetary union with an international goods market and incomplete international financial markets, in line with Quint and Rabanal (2014). Their financial accelerator mechanism differs from the one proposed by Bernanke et al. (1999), which I use in this model, because they abstract from asymmetric information. Thus, there is no default in their model unless borrowers find themselves completely underwater (that is, borrowers do not lie about their realized profits). However, this model presents asymmetric information, and therefore aggregate risk. This implies that financial intermediaries need to pay an auditing cost to verify that borrowers do not lie about their realized return. These monitoring costs result in a direct loss for the aggregate national output. In addition, the predetermined rate on loans included in Quint and Rabanal (2014) allows domestic financial intermediaries to obtain profits or losses. I simplify this assumption, even if it is less realistic, with a rate on loans that depends on the state of the economy so that domestic financial intermediaries deliver zero profits. This allows to characterize these agents as mere intermediaries between households (who lend funds) and entrepreneurs (borrowers).

Quint and Rabanal (2014) study the effects of a risk shock and observe that the active monetary policy (based on the anti-inflationary monetary policy of the EMU) cannot contain the accelerator effects of the economy. This is why they introduce macroprudential policy. They propose two alternative ways of macroprudential instrument design: one that targets the credit-to-GDP ratio and another that reacts to changes in the nominal credit growth. They argue that macroprudential policy delivers economic stability. Thus, Quint and Rabanal (2014) study the use of macroprudential policy, however they do not consider its interaction with a fiscal rule. I focus my analysis on the effects

of macroprudential policy, but as a complement to fiscal policy driven by a government spending rule. The interaction between fiscal and macroprudential policies is important given that they represent the whole set of instruments on which national authorities can count in the context of a monetary union. Moreover, fiscal policy plays an important role in the model as it is responsible for the private-public debt channel through which the destabilizing effects of the risk shock are propagated to the economy. Nevertheless, even including this additional national tool (fiscal policy), I find that, after a financial shock that increases the level of credit risk in a country of a monetary union, macroprudential instruments are needed. They complement traditional policies in the pursue of financial and economic stability because fiscal policy cannot attain this objective by itself.

The macroprudential instrument that I use is also based on Quint an Rabanal (2014) because it controls the amount of loans in the economy. I consider the nominal credit growth as the financial indicator. This is consistent with the Basel III broad macroprudential goal of protecting the banking sector from excessive credit growth. Basel III also states that "national authorities should monitor credit growth" and refers to it as an indicator that signals a build-up of system-wide risk. Monitoring credit growth is how macroprudential policy in the model pursues its objective of reducing macroeconomic and financial volatility.

As opposed to Quint and Rabanal (2014), I shed light on the differences between national versus supranational macroprudential policies. Other contributions to the debate on the desirability of implementing macroprudential policy at the union level or individually can be found within the recent macroprudential literature. Rubio (2014) analyzes the role of macroprudential policy in a heterogeneous monetary union comparing a scenario in which this policy is centralized against a scenario in which it is decentralized. The author concludes that the best option depends on the type of heterogeneity of the currency union. Rubio (2014) analyzes the effects of macroprudential policy when the union is hit by asymmetric technology shocks. However, my analysis revolves around the role of macroprudential policy after a credit risk shock, although I provide a robustness study for alternative asymmetric shocks. Conversely, Brzoza-Brzezina et al. (2015) also develop a two-country model for a monetary union and find that macroprudential policy can be viewed as a stabilizing tool only when it is implemented nationally. As opposed to them, I observe that a supranational macroprudential policy may stabilize one of the countries of the monetary union.

Dehmej and Gambacorta (2017) are part of the growing literature that states that monetary policy cannot lean against the wind to provide stability to the financial sector.

This statement is further reinforced by the situations in which asymmetric shocks hit the monetary union as monetary policy reacts only to average conditions. These authors also compare country-targeted macroprudential policy versus supranational macroprudential policy in a monetary union. They conclude that the former brings more advantages than the latter in terms of enhancing stability. My results imply that financial stability is stronger when macroprudential policy targets national variables but countries that are more destabilized after the shock can attain higher macroeconomic stability with a supranational macroprudential policy. Unlike Dehmej and Gambacorta (2017), I model imports and exports in the goods market and an international debt market which are determinant in the stabilization attained by macroprudential policy. But I coincide with them in the fact that national macroprudential policy is more appropriate than supranational macroprudential policy to stabilize the countries of a monetary union after asymmetric shocks. It seems unfair to implement the same supranational macroprudential policy targeting aggregate financial variables when not all the countries of the monetary union have the same needs.

Rubio and Carrasco-Gallego (2016) build a two-country model to compare the welfare gains when all the Euro Area countries coordinate in the implementation of national macroprudential policies to the welfare gains of the case in which there is no coordination. In the non-coordination case, a country does not implement macroprudential policy. They define macroprudential coordination as the situation in which each member state applies an equivalent macroprudential policy to that set by the others, responding to its own credit variables. The authors find that macroprudential policy always delivers financial stability but there are more welfare gains when all countries coordinate than when one country does not implement macroprudential policy. However, when there is no coordination, the country that does not coordinate benefits from its partners' policy implementation, due to a more stable financial system than when there is no macroprudential policy in place. Similarly to these authors, I also shed some light on the implications of a non-coordinated macroprudential scenario that can lead one country to free-ride. However, I consider that economies and shocks are not symmetric so my results go further. I find that the spillover effects of macroprudential policy to the country that does not implement it depend on whether this country is the one responsible for the financial shock or not. This implies that not all countries can free-ride because, in the event of asymmetric shocks, the country in which the financial shock is originated is stabilized only if it implements macroprudential policy. Indeed, in the model, this country is not affected by what the other country does, so it is the only one that can mend its unstable situation. The only

possible free-rider in this context is the country not responsible for the financial shock because it attains financial stability when the other country implements macroprudential policy[5].

Regarding the role of macroprudential instruments in the policy mix, there is an extensive literature that analyzes the interaction between monetary and macroprudential policies. Angelini et al. (2012) argue that, under financial shocks, the macroprudential and monetary policy coordination brings more macroeconomic stability than a "monetary-policy-only" scenario. Quint and Rabanal (2014) conclude that macroprudential policy reduces macroeconomic volatility and supports monetary policy by requiring smaller responses of the interest rate. I share with them the view that macroprudential policy is necessary when monetary policy cannot stabilize so effectively the financial system. But unlike all of them I add fiscal policy to the analysis of the policy mix.

Despite the extensive literature on the interaction of macroprudential and monetary policies, there is a scarce literature about how to coordinate macroprudential and fiscal policies, to which this paper also contributes. Claessens (2014) mentions the importance of coordinating macroprudential actions with other policies, such as fiscal or microprudential. Regarding fiscal policy and according to this author, some tax policies can contribute to systemic risk by encouraging private leverage, so macroprudential authorities need to coordinate with fiscal authorities. My work sheds new light on policy mix coordination: I show that in a monetary union where monetary policy cannot be used by national authorities, fiscal policy cannot stabilize public and private debt at the same time. This is why the following sections explore different ways of implementing a new tool, macroprudential policy, and how it interacts with the fiscal and monetary instruments in place.

4 The model

Based on the closed economy model of Fernández-Villaverde (2010a), I lay out a two-country DSGE model for a monetary union with financial frictions, as in Bernanke et al. (1999). The model includes an international financial market and a market for consumption goods that are internationally traded. Capital and labor are non-mobile across the two countries. The home country is of size n and the foreign country of size $1-n$. Each economy is composed of households, intermediate good producers, final good producers,

[5]Appendix E includes an analysis to compare the effects of macroprudential policy implemented coordinately versus macroprudential policy implemented only in one of the countries of the monetary union.

entrepreneurs, capital goods producers and domestic financial intermediaries. There is a single monetary authority for the currency union, while fiscal and macroprudential policies are implemented individually by national authorities. To model the international financial market I follow Quint and Rabanal (2014) and I include international financial intermediaries that connect the domestic financial intermediaries of both countries. De Blas and Malmierca (2019) provides a detailed explanation of the model, describing the problem of each agent for the home country. Moreover, in this research, the same maximization and minimization problems are applied to the foreign country. Variables and parameters for the foreign country are denoted with superscript $*$. The following sections and subsections contain an explanation of the open economy dimensions and the macroprudential policy related issues.

4.1 Households

There is a continuum of households with infinite life. The representative household maximizes its utility function, choosing total consumption, c_t, time devoted to work, l_t, and financial assets which are deposits, a_t, and government bonds, d_t, both in positive amounts.

Consumption by domestic households is composed by domestic goods and foreign goods in the form of imports. The domestic consumption index follows the form:

$$c_t = \left[(1-\varphi)^{\frac{1}{\zeta}} (c_{H,t})^{\frac{\zeta-1}{\zeta}} + \varphi^{\frac{1}{\zeta}} (c_{F,t})^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \tag{1}$$

where $c_{H,t}$ is the consumption of domestic goods and $c_{F,t}$ is the amount of imports. The parameter $\varphi \in [0,1]$ is a measure of the degree of openness, thus $1-\varphi$ represents the home bias in consumption. The degree of substitutability between domestic and foreign goods is given by $\zeta > 0$. Total consumption expenditures are

$$p_t c_t = p_{H,t} c_{H,t} + p_{F,t} c_{F,t}, \tag{2}$$

where the home consumer price index, p_t is composed by the price of domestic goods, $p_{H,t}$, and the price of foreign goods, $p_{F,t}$. For simplicity, I assume that the law of one price holds, therefore the prices of the goods produced at the foreign country are the same across countries and so are the prices of the goods produced at the home country.[6] That

[6] As this model represents a monetary union all prices are expressed in the same monetary units.

is, $p_{H,t} = p^*_{H,t}$ and $p_{F,t} = p^*_{F,t}$:

$$p_t = \left[(1-\varphi) (p_{H,t})^{1-\zeta} + \varphi (p_{F,t})^{1-\zeta} \right]^{\frac{1}{1-\zeta}}. \tag{3}$$

Domestic households choose their allocations between home and foreign goods maximizing their consumption (Equation (1)) subject to total expenditures. The demand equations for $c_{H,t}$ and $c_{F,t}$ can be derived from this maximization problem:

$$c_{H,t} = \left(\frac{p_{H,t}}{p_t} \right)^{-\zeta} (1-\varphi) c_t, \tag{4}$$

and

$$c_{F,t} = \left(\frac{p_{F,t}}{p_t} \right)^{-\zeta} \varphi c_t. \tag{5}$$

The same maximization problem applies for the foreign country with a degree of openness of φ^*.

The terms of trade are given by:

$$t_t = \frac{p_{F,t}}{p_{H,t}}. \tag{6}$$

This equation implies that an increase of t_t reflects a depreciation of the terms of trade and an increase of the competitiveness of domestically produced goods with respect to the goods produced in the foreign country.

4.2 International financial intermediaries

Following Quint and Rabanal (2014), the model incorporates intermediaries between domestic financial intermediaries of the home country and domestic financial intermediaries of the foreign country: international financial intermediaries. These agents borrow from the country with excess loanable funds to lend them to the country that has a shortage of loanable funds. They pay to the lending country a rate equal to the interest on deposits of that country and receive from the borrowing country a rate equal to the interest on deposits of that other country. Incomplete markets in this model imply that the interest rate differs across countries. Thus, the differential between the deposit interest rates of both countries equals the profits made by international financial intermediaries.[7] This

[7]Schmitt-Grohé and Uribe (2002) propose different alternatives to induce stationarity in a small open economy model with incomplete asset markets. In this line, I introduce an interest rate that is increasing

differential, also known as country debt premium, is given by

$$R_t - R_t^* = \kappa_t e^{\Omega\left(\frac{B_t}{p_t y} - \frac{B}{py}\right)} - 1. \tag{7}$$

For simplicity, as in Quint and Rabanal (2014), I take the home country as the reference so that the debt premium depends on the ratio of real international debt, $\frac{B_t}{p_t}$, to steady state real GDP, y, of the home country. In what follows I will denote real international debt by $\bar{B}_t$ and real private debt by $\bar{b}_t$. If the home country borrows from the international market I get $B_t > 0$ and then $R_t > R_t^*$. The parameter $\Omega > 0$ denotes the elasticity of the debt premium, and κ_t is a debt premium shock that follows

$$\kappa_t = \rho_\kappa \kappa_{t-1} + \sigma_\kappa \varepsilon_{\kappa,t}, \tag{8}$$

where $\rho_\kappa \in [0,1]$ is the persistence parameter; and σ_κ is the volatility of the shock, $\varepsilon_{\kappa,t} \backsim N(0,1)$.

Profits obtained by international financial intermediaries are distributed porportionally across households of both countries. Assuming that $nB_t = (1-n)(-B_t^*)$, international financial intermediaries receive:

$$R_t n B_t \; - \; R_t^* (1-n)(-B_t^*) \quad = \quad (R_t - R_t^*) n B_t \quad = \quad \kappa_t \left(e^{\Omega\left(\frac{B_t}{p_t y} - \frac{B}{py}\right)} - 1 \right) n B_t. \tag{9}$$

4.3 Fiscal authority

There is a national fiscal authority (or government) that finances its expenditures via taxes and public debt, according to the following budget constraint:

$$\frac{d_t}{p_t} = g_t + R_{t-1}^d \frac{d_{t-1}}{p_t} - tax_t, \tag{10}$$

where d_t denotes current issue of public debt; g_t is government spending; and tax_t are tax revenues defined by

$$tax_t = \tau_c c_t + \tau_l w_t l_t + \tau_R (R_{t-1} - 1) \frac{a_{t-1}}{p_t}. \tag{11}$$

As in Fernández Villaverde (2010), I assume that government spending evolves by the following fiscal rule:

$$\widehat{g}_t = \gamma_g \widehat{g}_{t-1} + d_g \frac{d_{t-1}}{p_t y_t} + \sigma_g \varepsilon_{g,t}, \tag{12}$$

in the level of debt.

where $\varepsilon_{g,t} \backsim N(0,1)$; $\widehat{g}_t$ are the log deviations with respect to the mean of the government spending process; and $d_g \leq 0$ is the sensitivity of government expenditure to changes in the ratio of debt over output, its sign reflects the objective of public debt stabilization. Parameter $\gamma_g \in [0,1]$ is the persistence coefficient and σ_g is the volatility of the government spending shock.

4.4 Monetary authority

The monetary authority or central bank is common for both countries and uses monetary policy to stabilize the monetary union gross inflation rate, Π_t^{MU}, and real output, y_t^{MU}. With that aim, the central bank sets the monetary policy instrument, or interest rate for the union. This analysis takes into account the active/passive definitions introduced by Leeper (1991). Leeper explains that an active policy is the one unconstrained by sovereign debt and a passive policy is the one constrained by current budgetary conditions and active authority actions. I consider the scenario where different national passive fiscal policies are combined with a single active monetary policy that stabilizes inflation at the union level.

Monetary union inflation is given by

$$\Pi_t^{MU} = \frac{p_t^{MU}}{p_{t-1}^{MU}}, \tag{13}$$

where

$$p_t^{MU} = (p_t)^n \, (p_t^*)^{1-n}, \tag{14}$$

and monetary union real output is

$$y_t^{MU} = (y_t)^n \, (y_t^*)^{1-n}. \tag{15}$$

The central bank follows a standard Taylor Rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left(\left(\frac{\Pi_t^{MU}}{\Pi^{MU}}\right)^{\gamma_\Pi} \left(\frac{y_t^{MU}}{y^{MU}}\right)^{\gamma_y}\right)^{(1-\gamma_R)} \exp(\sigma_m \varepsilon_{m,t}), \tag{16}$$

where $\gamma_R \in [0,1]$ is the persistence parameter; $\gamma_\Pi \geq 0$ and $\gamma_y \geq 0$ indicate how strong is the response of the interest policy rate to deviations of Π_t^{MU} and y_t^{MU} from their steady states, respectively; and σ_m is the volatility of the monetary policy shock, $\varepsilon_{m,t} \backsim N(0,1)$.

The nominal interest rate is modified through open market operations financed by

transfers, T_t and T_t^* for the home and foreign country, respectively.

4.5 Macroprudential authority

In this section, I include a macroprudential authority that sets policies to stabilize the financial system. Through macroprudential instruments the amount of loans to be lent to the private financial sector is controlled and private debt volatility is reduced in order to guarantee a more stable cycle.

Therefore, following Quint and Rabanal (2014), I introduce a macroprudential tool that controls the ability to lend of the domestic financial intermediaries in the following way:

$$\frac{1}{\eta_t}\left(B_t + a_t\right) = b_t, \tag{17}$$

where η_t is a new variable that affects the credit market conditions.

The macroprudential regulation will affect financial variables countercyclically. Higher values of η_t reflect a tightening macroprudential policy, while lower values reflect an easing macroprudential policy. This macroprudential rule implies that, when the regulation is tightening, domestic financial intermediaries can only lend a fraction of the funds they get from households and from international financial intermediaries. In this case, this measure would be equivalent to a reserve requirement ratio. However, in line with Quint and Rabanal (2014), I allow the macroprudential instrument to behave symmetrically and go below one. Thus, when the regulation is easing, the central bank will provide liquidity to domestic financial intermediaries so that they can lend more than the amount of deposits and international funds they hold on their balance sheet.

In line with Quint and Rabanal (2014), I also make η_t dependent on the deviation of credit market conditions, Ψ_t, from their steady state, Ψ, as follows:

$$\eta_t = \left(\frac{\Psi_t}{\Psi}\right)^{\gamma_\eta}, \tag{18}$$

where $\gamma_\eta > 0$ reflects how responsive η_t is to the indicator of credit market conditions considered. Notice that macroprudential policies do not affect the steady state since $\eta = 1$ whenever $\Psi_t = \Psi$.

De Blas and Malmierca (2019) includes an analysis of the two alternative macroprudential instruments proposed by Quint and Rabanal (2014). They first define Ψ_t as the deviation of the nominal private credit growth and second as the deviation of the private credit-to-GDP ratio.

The results obtained in de Blas and Malmierca (2019), for a closed economy, show that macroprudential policy always stabilizes private debt but GDP only when it targets nominal credit growth. As my objective is to analyze macroprudential policy as a way of attaining macroeconomic and financial stability, in this paper I define Ψ_t as the nominal private credit growth. This is consistent with Basel III that states that monitoring excessive credit growth is one of the most important financial indicators that should be consider when implementing macroprudential policy. Therefore,

$$\Psi_t = \frac{\bar{b}_t}{\bar{b}_{t-1}}\Pi_t. \tag{19}$$

Thus, the macroprudential instrument becomes tightening when there is an increase in the nominal private credit growth and easing if the latter decreases.

As in Dehmej and Gambacorta (2017), I analyze the case of supranational macroprudential policy. The macroprudential tool is the same in both countries of the union and it targets aggregate nominal credit growth with a degree of responsiveness of γ_η^{MU}, therefore:

$$\eta_t^{MU} = \left(\frac{\Psi_t^{MU}}{\Psi^{MU}}\right)^{\gamma_\eta^{MU}}, \tag{20}$$

being

$$\Psi_t^{MU} = \frac{\bar{b}_t^{MU}}{\bar{b}_{t-1}^{MU}}\Pi_t^{MU}, \tag{21}$$

where $\bar{b}_t^{MU}$ is the aggregate real private debt,

$$\bar{b}_t^{MU} = \left(\bar{b}_t\right)^n \left(\bar{b}_t^*\right)^{1-n}, \tag{22}$$

and with Π_t^{MU} denoting the monetary union inflation.

The introduction of macroprudential policy affects the credit conditions in the model.[8] In particular, the lending-deposit spread becomes

$$\frac{R_{t+1}^l}{R_t} = \frac{s_t\eta_t}{\left[1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] + \frac{(1-\mu)}{\varpi_{t+1}}\int_0^{\varpi_{t+1}} \omega dF\left(\omega, \sigma_{\omega,t}\right)} \tag{23}$$

When the macroprudential policy is tightening, the lending-deposit spread increases. That is, a tightening macroprudential policy means less funds are available to lend, without any change in the policy rate, widening the gap between the lending and the deposit

[8] A detailed explanation can be found in Appendix D.

rates. The opposite holds when the macroprudential policy is easing.

The one period interest rate of the loan is set on the contract that the domestic financial intermediary agrees with the entrepreneur. The previous expression shows that R^l_{t+1} also depends on the level of η_t for the current period, so the macroprudential policy affects the contractual agreement. In particular, when the macroprudential rule is too restrictive the R^l_{t+1} set in the contract is higher than when macroprudential policy is relaxed. This ensures that when macroprudential policy is introduced, domestic financial intermediaries can still obtain zero profits, paying the same R_t to households and international financial intermediaries. The rate on loans, R^l_{t+1}, is the only one affected by macroprudential policy. Therefore, despite macroprudential policy, lending funds in the form of deposits or through an international bond to financial intermediaries is still worth it for households and international intermediaries. Entrepreneurs, however, face a higher cost on their debt if they need to borrow when macroprudential policy is tightening, and vice versa. As a consequence private credit is affected not only from the supply side but also from the demand side, which is the goal of macroprudential policy.

5 Aggregation and Equilibrium

Aggregate demand in the model is given by

$$y_t = c_{H,t} + \frac{1-n}{n} c^*_{H,t} + i_t + g_t + \mu G\left(\varpi_t, \sigma_{\omega,t-1}\right)\left(r_t + q_t\left(1-\delta\right)\right) k_{t-1}, \tag{24}$$

for the home country. And the aggregate supply for the home country is

$$y_t = \frac{1}{\upsilon_t} e^{z_t} k^{\alpha}_{t-1} l^{1-\alpha}_t, \tag{25}$$

being υ_t the inefficiency created by price dispersion that evolves as:

$$\upsilon_t = \theta \left(\frac{\Pi^{\chi}_{H,t-1}}{\Pi_{H,t}}\right)^{-\varepsilon} \upsilon_{t-1} + \left(1-\theta\right)\left(\bar{\Pi}_{H,t}\right)^{-\varepsilon}. \tag{26}$$

The home country's net foreign asset position is

$$n\bar{B}_t = nR_{t-1}\frac{\bar{B}_{t-1}}{\Pi_t} + n\frac{p_{F,t}}{p_t} c_{F,t} - \left(1-n\right)\frac{p_{H,t}}{p_t} c^*_{H,t}. \tag{27}$$

The equilibrium in this model, considering that there is a home country and a foreign

country, can be defined as the sequence of quantities $\{c_t,\, c_{H,t},\, c_{F,t},\, l_t,\, a_t,\, k_t,\, i_t,\, b_t,\, B_t,\, c_t^*,\, c_{H,t}^*,\, c_{F,t}^*,\, l_t^*,\, a_t^*,\, k_t^*,\, i_t^*,\, b_t^*,\, B_t^*\}_{t=0}^{\infty}$; fiscal policy $\{g_t,\, tax_t,\, d_t,\, g_t^*,\, tax_t^*,\, d_t^*\}_{t=0}^{\infty}$; prices $\{p_t,\, p_{H,t},\, p_{F,t},\, r_t,\, w_t,\, q_t,\, p_t^*,\, r_t^*,\, w_t^*,\, q_t^*\}_{t=0}^{\infty}$, and interest rates $\{R_t^d,\, R_t,\, R_t^k,\, R_t^l,\, R_t^{d*},\, R_t^*,\, R_t^{k*},\, R_t^{l*}\}_{t=0}^{\infty}$, given exogenous variables $\{z_t,\, \widehat{\sigma}_{\omega,t},\, \widetilde{s}_t,\, \phi_t,\, z_t^*,\, \widehat{\sigma}_{\omega,t}^*,\, \widetilde{s}_t^*,\, \phi_t^*,\, \kappa_t\}_{t=0}^{\infty}$, such that:

- optimization problems are satisfied for all agents of both countries in the model; and

- all markets clear, that is, in the case of the home country

$$y_t = c_{H,t} + \frac{1-n}{n} c_{H,t}^* + i_t + g_t + \mu G\left(\varpi_t, \sigma_{\omega,t-1}\right)\left(r_t + q_t\left(1-\delta\right)\right) k_{t-1},$$

$$y_t = \frac{1}{\upsilon_t} e^{z_t} k_{t-1}^{\alpha} l_t^{1-\alpha},$$

$$l_t^s = l_t^d,$$

$$nB_t = \left(1-n\right)\left(-B_t^*\right)$$

$$\begin{cases} a_t + B_t = b_t & \text{if macroprudential policy is not included,} \\ \frac{1}{\eta_t}\left(a_t + B_t\right) = b_t & \text{if macroprudential policy is included.} \end{cases}$$

- plus the laws of motion

$$k_t = (1-\delta)k_{t-1} + \left(1 - S\left[\frac{i_t}{i_{t-1}}\right]\right) i_t, \text{and}$$

$$\frac{d_t}{p_t} = g_t + R_{t-1}^d \frac{d_{t-1}}{p_t} - tax_t.$$

$$n\bar{B}_t = nR_{t-1}\frac{\bar{B}_{t-1}}{\Pi_t} + n\frac{p_{F,t}}{p_t} c_{F,t} - \left(1-n\right)\frac{p_{H,t}}{p_t} c_{H,t}^*.$$

For the foreign country the market clearing is replicated in the same way but using the foreign variables of the model.

6 Calibration of the parameters and steady state

Table 2 shows the parametrization I use in the model. I calibrate most of the parameters based on Gomes and Seoane (2018), Fernández-Villaverde (2012), Fernández-Villaverde

(2010b) or Bernanke et al. (1999). All parameters and steady states are the same for both countries except for home country imports and foreign country imports, $\frac{c_F}{y}$ and $\frac{c_H^*}{y^*}$ respectively, and the steady states that result from these values. Also γ_η and γ_η^* depend on the macroprudential scenario considered. Table 3 includes a summary of the steady state values that are relevant for the analysis.

Open economy. I assume that both countries are of equal size, $n = 0.5$. Then I set the fraction of imported goods from the foreign country to the home country over GDP to 0.1 and the fraction of imported goods from the home country to the foreign country over foreign GDP to 0.11. Therefore, the home country is a net exporter and the foreign country a net importer in steady state what, taking into account the net foreign asset position, implies that international debt is different from 0. The substitutability between domestic and foreign goods is set to $\zeta = 1.5$. The terms of trade, t, are 1 in steady state what means that the price of the goods produced in the home country is the same as the price of the foreign country produced goods. The debt elasticity of the country premium is different to zero to induce stationarity (Schmitt-Grohé and Uribe, 2002), concretely $\Omega = 0.0043$.

Preferences. I set the discount factor to $\beta = 0.999$, being the same for both countries, and $\Pi_H = \Pi_F = \Pi = \Pi^* = 1.005$, implying an average annual real interest rate equal to 0.4%. Habits on consumption are $h = 0.5$, and the Frisch elasticity of labor is $1/\vartheta = 2$. Labor in steady state is $l = \frac{1}{3}$.

Technology. The capital share, α, is set equal to 0.33; capital depreciation rate, δ, equals 8.9% at an annual rate; and capital adjustment costs are such that $S^{"}[1] = 14.477$. The Calvo pricing parameter, θ, is 0.8 what means on average 5 quarters of duration of prices; the degree of indexation to past inflation, χ, equals 0.6; and the elasticity of substitution across goods, $\varepsilon = 8.577$, implies a markup of around 13% in the goods sector.

Financial variables. I consider monitoring costs, μ, are 15% of the entrepreneur's output; the loan-to-capital ratio is set equal to $\frac{\bar{b}}{k} = \frac{1}{3}$; the average spread on loans, s, is 0.25%; the survival rate of entrepreneurs is $\gamma^e = 0.975$; and the annual probability of default is 3%.

Fiscal policy. The steady state values for the tax rates are $\tau_l = 0.24$ and $\tau_r = 0.42$; government spending-to-GDP ratio equals 20%, and the debt-to-GDP ratio is 60%. Given these values τ_c is determined from the government's budget constraint. Parameter d_g in the fiscal rule is set to -0.01, meaning that the government spending rule is set to stabilize public debt and thus fiscal policy used in this model is passive.

Monetary policy. In the analysis below, monetary policy is conducted at the union level.

I assume that the response of intervention rate to changes in inflation is $\gamma_{\Pi}(1-\gamma_R)=1.5$ what implies that the monetary union authorities have the objective of inflation stabilization, so monetary policy is active.

Macroprudential policy. The macroprudential policy parameters, γ_η and γ_η^*, equal 0 when there is no macroprudential policy and are set to 1.75 when macroprudential policy is included.

Shock processes. I consider quite permanent shock processes, therefore, I set autoregressive coefficients equal to 0.95, and standard deviations are taken from the empirical evidence and past literature, as summarized in Table 2.

7 Impulse Response Functions (IRFs)

This section analyzes the response of the main economic variables of the two-country model to a credit risk shock originated in the home country. In all the scenarios considered the shock is a 1 percent standard deviation increase in private credit risk, $\sigma_{\omega,t}$. Following Leeper's definition, monetary policy is always active in this analysis. According to Leith and Wren-Lewis (2006), to attain a determinate equilibrium in a monetary union where the monetary authority targets inflation, each nation of the union needs to stabilize its public debt through fiscal policy. In line with Leeper (1991), this kind of fiscal policy is passive. This paper focuses on the use of a government spending rule and leaves aside the tax rule to isolate the effects of the former. In this economy, monetary policy aims at pursuing price stability for the monetary union. Therefore if it leans against the wind to solve financial stability problems it will leave aside its main objective. This implies that a new toolkit needs to be added to the financial stability framework: macroprudential policy.

The scenarios analyzed below compare a version of the model without macroprudential policy with two other versions implying different macroprudential policy implementations in a monetary union: country-targeted and supranational macroprudential measures. A country-targeted macroprudential policy is the one that reacts to national nominal private debt growth. A supranational macroprudential policy targets the growth of the aggregate nominal credit. In both scenarios all the countries belonging to the monetary union implement the corresponding macroprudential toolkit, that is, there is always coordination in the use of macroprudential policy[9].

[9]Appendix E includes two alternative country-targeted macroprudential scenarios, named non-coordinated macroprudential scenarios: in one of them the home country is the only one that applies

In what follows, I comment on the main differences between the three macroprudential scenarios represented in Figure 3 and Figure 4: no macroprudential (solid), country-targeted macroprudential (dashed) and supranational macroprudential (dotted).

In all the three scenarios, an increase in the credit risk of the home country private sector raises the probability of default of the home country entrepreneurs. Thus, home country lenders toughen the terms of the contract by increasing the interest rate paid on loans (not shown in the figures). This generates a decrease in the demand of private debt that brings down the demand for capital and its price, the Tobin's q, (neither of them shown in the figures). The Tobin's q values the firm's assets, thus the firm's networth in the home country is also reduced. Quint and Rabanal (2014) analyze the effects of a negative credit risk shock that reduces risk in the country where it is originated. They also find that private debt moves inversely with respect to the probability of default.

If there is no macroprudential policy, the effects just described are larger for the financial sector, implying a sharp decrease in private investment and making GDP fall in the home country. This is in line with Christiano et al. (2010), Gomes and Seoane (2018) and de Blas and Malmierca (2019). The fall in investment and private debt in the home country also reduces its demand for international funds (the home country is a net international borrower in the steady state), thus putting down the risk premium on international debt. This implies a reduction in the differential between the deposit rates of both countries. As a consequence of the home country recession, in the absence of macroprudential policy, there is a capital flight from the latter to the foreign country. The capital flight generates higher levels of foreign private debt and foreign investment, increasing foreign output.

Due to the fall of public revenues, public debt rises as output goes down in the home country, in the no macroprudential scenario. The opposite happens in the foreign country.

macroprudential policy and in the other macroprudential policy is implemented only by the foreign country. The IRFs for these alternative non coordinated scenarios are plotted in Figure 5 and Figure 6. The home country is not affected by what the foreign country does, so for the home country the former scenario is equivalent to the case in which both implement macroprudential policy and the latter scenario to the no macroprudential policy case. The foreign country however is more stabilized when the home country applies macroprudential policy than when it is implemented by itself or by both countries at the same time. These results provide a rationale for the incentives that the country not responsible for the risk shock has to free-ride on the macroprudential policy of the other country. Moreover, this is consistent with the findings of Rubio and Carrasco-Gallego (2016) about the stabilization benefits for a country that does not implement macroprudential policy when the other countries of the same monetary union implement it. However, my results show that, unlike what Rubio and Carrasco-Gallego (2016) find, the country where the shock is originated cannot benefit from the other country's macroprudential policies. The main reason for this difference is that they consider that all countries are hit by symmetric shocks.

Therefore, in the home country lower private debt depresses investment and output so tax collection falls pushing upwards public debt, that is, generating the private-public debt channel. In the foreign country, as a consequence of the home country credit risk shock, entrepreneurs expand their private investment resulting in a rise of output and private leverage. Foreign labor goes up increasing the collection of taxes during the first periods that immediately follow the shock. It is remarkable that around the fifth period after the shock, the collection of foreign taxes starts to decrease as a consequence of a fall in the return on deposits. The return on deposits is directly related to the policy rate that, in this scenario, goes down to stabilize the monetary union aggregate inflation. Both the rise of tax revenues and inflation (debt deflation channel) bring down foreign public debt immediately after the shock, activating the private-public debt channel also in the foreign country. Thus, for the model and calibration used in this research, the private-public debt channel operates in both countries when macroprudential policy is not implemented.

However, macroprudential policy changes significantly the effects of the credit risk shock. As mentioned in Section 4.5, the analysis is focused on the implementation of a macroprudential tool, η_t, that reacts to nominal credit growth. In the home country, after the credit risk shock that brings down private leveraging, a countercyclical macroprudential policy eases credit conditions with respect to the no macroprudential scenario, alleviating the fall of private debt. Therefore, the decrease of investment in response to the shock is smoothed. This is passed on to output and, thus, to public revenues. In the foreign country however, very different effects arise from the application of either national or supranational macroprudential policies, which are discussed below.

7.1 Implementation of national macroprudential policy

When macroprudential policy targets the growth of national nominal credit, the private-public debt channel is offset and macroeconomic and financial stabilization is achieved for both countries (see the volatilities in Table 4 and correlations in Table 5). A weaker fall of private debt in the home country together with the improvement of financial conditions bring the home country output to a more stable path, with respect to the no macroprudential scenario. Foreign private debt still increases smoothly in this scenario but is almost isolated from the effects of the shock. This time, the fall of foreign networth and Tobin's q makes foreign investment fall down slightly so foreign output also experiences a smooth decrease. Foreign output is also more stable due to a foreign macroprudential policy that restricts credit conditions and refrains investment and GDP growth. In both countries, automatic stabilizers transfer the effect of output stabilization to public debt. Moreover,

there is a debt deflation effect by which as inflation goes up real public debt decreases in the home country and as inflation falls real public debt rises in the foreign country. Therefore, by stabilizing private and public debt at the same time, the country-targeted macroprudential policy offsets the private-public debt channel in both countries.

Nevertheless, there is an additional channel that contributes to stabilize home country and foreign GDP when the macroprudential policy is country-targeted: the open economy channel. This channel compensates the countercyclical effects of macroprudential policy on home and foreign GDP, operating in the following way. Home country inflation increases and foreign inflation goes down so the terms of trade decrease, that is, foreign goods are more competitive than home country goods because their relative price is lower. Then, the home country increases imports and decreases exports (i.e, net exports go down). This is consistent with the expenditure-switching effect to which some traditional open macro models refer (see Engel, 2003; Galí and Monacelli, 2003; or Corsetti, 2007). The expenditure switching effect compensates partially the effects of investment on output in both countries, contributing to macroeconomic stabilization.

In line with these results, recent literature on macroprudential policy also finds that the introduction of the latter reduces macroeconomic and financial volatility (see for example Quint and Rabanal, 2014; Rubio and Carrasco-Gallego, 2016; or Dehmej and Gambacorta, 2017).

7.2 Implementation of supranational macroprudential policy

The above mentioned open economy channel explains why, when countries implement a supranational macroprudential policy, the home country experiences strong macroeconomic and weaker financial stabilization. At the same time, this channel is responsible for the foreign country destabilization in terms of output, public and private debt (see Table 4 for the main variables' volatility). It is worth emphasizing that, in this scenario, stabilization in the home country is not due to the cancellation of the private-public debt channel as the latter remains at work under supranational macroprudential policy. As this kind of macroprudential policy does not target national variables directly, the decrease of home country private debt after the shock is smoothed but not as much as in the national macroprudential case. Thus, the private debt behavior fails to increase investment in the home country, although the latter falls by less than in the no macroprudential situation.

Patently, the effects of supranational macroprudential policy on national variables are not as countercyclical as when a national macroprudential policy is in place. Instead, it is possible to observe how the foreign macroprudential tool reacts much more than what

its economy needs. This supranational policy encourages significantly foreign private debt, what is translated into more foreign investment and a sharp growth of foreign GDP. Table 4 shows how foreign output is destabilized with this kind of supranational policy.

As mentioned before, in the supranational macroprudential scenario, the open economy channel that contributes to output stabilization in the home country. Foreign inflation goes up and home country inflation decreases what raises the terms of trade. The open economy channel appears because the rise of the terms of trade increases home country net exports. Then, this channel compensates the effect of the home country investment fall, moderating the path of the home country GDP. The role of proportional taxes in this supranational scenario is crucial for both countries, being the main responsible for the private-public debt channel. In the home country, tax revenues go down following a similar path to output so public debt rises moderately. So government debt increases as private debt falls, thus the channel is still at work. In the foreign country, the rise of GDP increases tax revenues what provokes a deep decrease of public debt. As foreign private debt grows with this supranational policy and public debt goes down the private-public debt channel also operates here.

7.3 Volatility and correlations

These results about stabilization are confirmed in Table 4. The correlation between the private debt-to GDP ratio and the public debt-to-GDP ratio[10] (see Table 5) suggests that the national macroprudential tool is the only one that offsets the private-public debt channel. It can also be observed that the negative correlation between private and public debt does not disappear when a supranational macroprudential policy is implemented. To summarize, the main reason for these results is that country-targeted macroprudential policy inverts the response of public debt after a financial shock. However, supranational macroprudential policy only manages to smooth public debt in the home country and it amplifies the effects of the shock on public debt in the foreign country.

There is a trade-off between stabilizing considerably the home country's GDP, which is the most destabilized after the shock, and stabilizing the foreign country's GDP. The country-targeted macroprudential scenario delivers financial and macroeconomic stability for both countries by offsetting the private-public debt channel. In this scenario, stabilization in the foreign country is significant due to the open economy channel. Dehmej and Gambacorta (2017) find more appropriate, in terms of optimality and stability, the

[10]I focus on these ratios to analyze a measure that is similar to the data that was collected in the empirical analysis of this paper.

implementation of macroprudential policies that target national variables. They base this argument on the fact that national macroprudential policies can neutralize the effects of asymmetric shocks while supranational macroprudential policies cannot. In line with these authors, I find that national macroprudential policy implementation is more appropriate given that it targets each country's specific needs. However, my analysis also shows some stabilization advantages of implementing supranational macroprudential policy. The latter generates an alternative channel for attaining macroeconomic stability in the home country, the open economy channel, but at the cost of destabilizing the foreign country.

Comparing these results to the ones obtained for a closed economy (see de Blas and Malmierca, 2019), in the open economy version of the model shows a more persistent response of output to a credit risk shock when macroprudential policy is implemented. The reason is the open economy channel generated by the international goods market of this model. When macroprudential policy is used, there is an increase on impact of domestic inflation and a decrease on impact of foreign inflation. However, after the immediate effect of the financial shock, home country inflation falls making the price of home goods more competitive relative to that of foreign goods. Then, the home country start to increase its net exports around the fifth period after the shock, what contributes to a more persistent rise of GDP.

Another interesting result from this analysis is that macroprudential policy, no matter how it is implemented, contributes to the monetary policy role of inflation stabilization and requires smoother responses of the policy rate. This result is also obtained by Quint and Rabanal (2014), as they conclude that macroprudential policy "lends a hand" to monetary policy. As a consequence of macroprudential policy implementation, inflation increases and so does the policy rate. Then, independently of the effects that macroprudential policy has on net exports, there is a fall in total consumption in both countries because the rise in the interest rate induces consumers to delay consumption.

8 Robustness analysis

The analysis carried out so far assumes that the economy is mostly affected by a credit risk shock, which seems to have dominated the years of the financial recession of 2007. Nevertheless, the economy can be hit by a variety of shocks, so the role of macroprudential policies must be analyzed outside the framework of credit risk shocks, to evaluate its effectiveness under other possible scenarios.

Therefore, in this section, I extend the analysis by looking at the stabilization properties of macroprudential policy in the event of alternative asymmetric shocks other than the credit risk shock. To that end, I consider that the home country is hit by either a spread shock, a preference shock or a technology shock.[11]

In all the three cases, macroprudential policies attain macroeconomic stabilization for the country that suffers the shock but macroeconomic destabilization for the foreign country. After a spread shock, the home country private debt is also stabilized when macroprudential policy is introduced, no matter how it is designed. The financial sector of the foreign country is stabilized when macroprudential policy targets national variables while it is destabilized when macroprudential measures target the average union variables. In the event of a preference shock, the home country attains more financial stability with macroprudential policy while the foreign country is financially destabilized with national macroprudential policy and financially stabilized with supranational macroprudential policy. Finally, when the home country is hit by a technology shock, the financial sector of the home country is destabilized with macroprudential policy but, this time, macroprudential measures stabilize the private debt of the foreign country. It is worth mentioning that supranational macroprudential policy always results in a more volatile scenario in both countries than the country-targeted macroprudential policy.

Tables 6, 7 and 8 contain the volatilities of the main economic variables for each shock explained above and the correlations between private and public debt. The tables show that after a spread shock, the private-public debt channel arises in both countries and macroprudential policy only manages to eliminate it in the foreign country. In the event of a technology shock, the channel, present in the home country, is offset when macroprudential policy is implemented nationally. Finally, when the economy is hit by a preference shock originated in the home country, the private-public debt channel appears in the foreign country and is eliminated with any kind of macroprudential policy.

As a conclusion, and in line with the previous literature (Angelini et al., 2012 or Quint and Rabanal, 2014), this analysis implies that macroprudential policy attains its stabilization objectives depending on the shock that hits the economy.

[11]Notice that to make the analysis more comparable I consider a magnitude for each shock that leads to the same volatility of the home country GDP in the no macroprudential scenario, $\sigma(GDP) = 0.0174$, the volatility implied by a credit risk shock with a standard deviation of 0.560 as estimated by Gomes and Seoane (2018).

9 Conclusion

This paper considers the implementation of macroprudential policy in a two-country model for a monetary union to complement fiscal policy when monetary policy cannot be used at the national level. With that aim, I compare two alternative ways of implementing macroprudential policy in a monetary union: a country-targeted tool versus a supranational tool.

There is a private-public debt channel that destabilizes the economy after a credit risk shock as in de Blas and Malmierca (2019). This analysis shows how the cancellation of this channel (through the implementation of macroprudential policy) can help stabilize the home and foreign economies. However, there is an additional channel that is crucial in this open economy model: the open economy channel. This channel can contribute to macroeconomic stabilization depending on how macroprudential policy is designed.

When macroprudential policy targets national financial variables, the private-public debt channel is offset so both countries in the union reach more financial and macroeconomic stabilization than in the no macroprudential case. The supranational macroprudential scenario maintains at work the private-public debt channel in both countries. The reason is that public debt increases in the home country and decreases in the foreign country, while private debts perform the opposite paths. However, due to the open economy channel, this macroprudential scenario is the one that brings the greatest macroeconomic stability for the home country, although at the same time, the home country private debt is not as stable as in the national macroprudential case. The destabilization of the foreign country in the supranational macroprudential scenario arises from the sharp increase of foreign private debt after the shock, consequence of a foreign easing macroprudential policy.

Therefore, in this model, when supranational macroprudential policies are implemented, the country responsible for the credit risk shock attains the greatest macroeconomic stability, at the cost of destabilizing the foreign country. On the other hand, national macroprudential policy brings lower levels of macroeconomic stability to the home country but it also provides with macroeconomic stability to the country not responsible for the shock, and that suffers the effects of its neighbor's behavior.

Macroprudential policy can either reduce or support credit growth, although, as explained by Cerutti et al. (2015), it works better in booms, i.e. constraining credit. But these measures are also operative in busts, i.e. limiting declines in credit. This implies that macroprudential instruments may need to react with a different intensity to encour-

age or limit credit. In line with this, an interesting analysis left for future research is the implementation of a state-contingent macroprudential policy that could adapt its degree of responsiveness to financial indicators, depending on the phase of the cycle.

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Appendix A. The data

Data

Data for Spain and Germany cover period 1960-2017 for the main series of interest. More concretely, data in Table 1 are: real government consolidated gross debt-to-real GDP ratio, D, real credit to the private non-financial corporations-to-real GDP ratio, B, real GDP, Y, and real government final consumption expenditure, G. Real GDP and the GDP deflator were collected from the European Commission's AMECO Database. Real public debt is the deflated series of the nominal general government consolidated gross debt obtained from AMECO. Data on real private debt was generated by deflating the nominal series available at the Bank of International Settlements (BIS) on credit, from all sectors of the economy, to the private non-financial sector (non financial corporations, households and non-profit institutions serving households), adjusted for breaks. Real public spending is the deflated series of OECD data on nominal total general government expenditure.

As in de Blas and Malmierca (2019), for the comparison of the data I detrend both the real GDP and the real public expenditure applying the Hodrick Prescott filter. To evaluate real private and public debt I use their ratio over GDP.

Appendix B. The model

B.1 Households

There is a continuum of households with infinite life. The representative household maximizes his utility function, choosing total consumption, c_t, of foreign or domestic goods, time devoted to work, l_t, and financial assets, deposits, a_t, and government bonds, d_t, both in positive amounts. The individual's utility function is given by

$$E_t \sum_{t=0}^{\infty} \beta^t e^{\phi_t} \left[\log\left(c_t - h c_{t-1}\right) - \psi \frac{l_t^{1+\vartheta}}{1+\vartheta} \right], \tag{B.1.1}$$

where $\beta \in (0,1)$ is the discount factor; $h \geq 0$ reflects the degree of habit persistence; $\psi > 0$ denotes the magnitude of the labor disutility relative to consumption utility; and $\vartheta > 0$ is the inverse of the Frisch elasticity of labor supply. Variable ϕ_t represents an intertemporal preference shock with law of motion

$$\phi_t = \rho_d \phi_{t-1} + \sigma_\phi \varepsilon_{\phi,t} \text{ where } 0 < \rho_d < 1 \text{ and } \varepsilon_{\phi,t} \backsim N(0,1). \tag{B.1.2}$$

Parameter ρ_ϕ is the persistence coefficient and σ_ϕ the volatility of the preference shock.

The household makes decisions subject to the following budget constraint:

$$(1+\tau_c)\,c_t + \frac{a_t}{p_t} + \frac{d_t}{p_t} = (1-\tau_l)\,w_t l_t + \left[1+(1-\tau_R)\,(R_{t-1}-1)\right]\frac{a_{t-1}}{p_t} + R^d_{t-1}\frac{d_{t-1}}{p_t} + T_t + F_t + tre_t. \quad \text{(B.1.3)}$$

The left hand side of equation (B.1.3) represents the household's expenditures in real terms. The right hand side describes the sources of income to the household: labor income, $w_t l_t$, where w_t is the real wage; interests on last period investment on deposits, $R_{t-1}a_{t-1}$ and on public assets, $R^d_{t-1}d_{t-1}$; and net transfers that households receive from the government, T_t. The model includes proportional taxes on real consumption, τ_c, on labor income, τ_l and on net returns on deposits, τ_R.[12] Dividends are paid by firms to households, F_t; and a net transfer that households receive from entrepreneurs, tre_t, defined as follows:

$$tre_t = (1-\gamma^e)\,n_t - w^e. \quad \text{(B.1.4)}$$

where $\gamma^e = \frac{1}{1+e^{-\bar{\gamma}^e}}$ is the rate of entrepreneurs that survives from one period to the next one. Then the net wealth of the dead entrepreneurs, $\left(1-\frac{1}{1+e^{-\bar{\gamma}^e}}\right)n_t$, is paid back to households and these transfer w^e to incoming entrepreneurs. This constitutes the initial real net wealth of the new entrepreneurs.

The first order conditions obtained from the representative household's problem are

$$e^{\phi_t}\frac{1}{c_t - hc_{t-1}} - \beta E_t \frac{h}{c_{t+1} - hc_t} = \lambda_t\,(1+\tau_c)\,, \quad \text{(B.1.5)}$$

$$\lambda_t = \beta E_t \lambda_{t+1}\frac{\left[1+(1-\tau_R)\,(R_t-1)\right]}{\Pi_{t+1}}, \quad \text{(B.1.6)}$$

$$\lambda_t = \beta E_t \lambda_{t+1}\frac{R^d_t}{\Pi_{t+1}}, \quad \text{(B.1.7)}$$

$$e^{\phi_t}\psi l_t^{\vartheta} = (1-\tau_l)\,w_t\lambda_t, \quad \text{(B.1.8)}$$

where λ_t is the Lagrange multiplier that represents the marginal value of wealth of house-

[12] Returns on sovereign debt are not taxed because, as Fernández-Villaverde (2010) says, otherwise the government would have to pay a higher interest rate on public debt to compensate for the lower net return that households would receive due to the tax, thus the effect would be the same.

holds.

Consumption by domestic households is composed by domestic goods and foreign goods in the form of imports.

B.2 Intermediate goods producers

These agents produce differentiated goods that are then sold in a monopolistically competitive market to final good producers, who use them in their production process. Each intermediate good producer, i, chooses labor l_{it} and capital k_{it-1} as factors of production and they create their output y_{it} through the following constant returns to scale Cobb-Douglas production function:

$$y_{it} = e^{z_t} k_{it-1}^{\alpha} l_{it}^{1-\alpha}, \tag{B.2.1}$$

where $0 \leq \alpha \leq 1$ is the capital share of the intermediate production function.

Technology follows an exogenous AR(1) process $z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}$ where $0 < \rho_z < 1$ and $\varepsilon_{z,t} \backsim N(0,1)$, being ρ_z the persistence coefficient and σ_z the volatility of the technology shock.

Labor is hired from households in exchange for real wages w_t. Capital is rented from entrepreneurs at a real interest rate r_t. Cost minimization implies

$$k_{it-1} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t} l_{it} \frac{p_t}{p_{H,t}}. \tag{B.2.2}$$

These firms reset their prices through a Calvo pricing mechanism. Each period, a fraction $1-\theta$ of producers can change their price, while a fraction θ has to keep the previous period's price which is then indexed to past inflation.

Firms resetting their price in period t maximize the following expression:[13]

$$E_t \sum_{\tau=0}^{\infty} (\beta\theta)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left[\left(\prod_{s=1}^{\tau} \frac{\Pi_{H,t+s-1}^{\chi}}{\Pi_{H,t+s}} \frac{p_{H,it}}{p_{H,t}} - mc_{t+\tau} \right) y_{it+\tau} \right], \tag{B.2.3}$$

subject to a sequence of demand functions

$$y_{it+\tau} = \left(\prod_{s=1}^{\tau} \frac{\Pi_{H,t+s-1}^{\chi}}{\Pi_{H,t+s}} \frac{p_{H,it}}{p_{H,t}} \right)^{-\varepsilon} y_{t+\tau}. \tag{B.2.4}$$

[13] The expression represents the discounted sum of the difference between the optimizing firm's revenues and its marginal cost, that is, the discounted profits.

In the expressions above, $\frac{\lambda_{t+\tau}}{\lambda_t}$ is the stochastic discount factor, taken as given by the monopolistically competitive firm; mc_t denotes the marginal cost of the intermediate good producer; $p_{H,it}$ is the price set in period t by the domestic intermediate firm i; $p_{H,t}$ is the aggregate domestic price level; $\Pi_{H,t}$ denotes domestic inflation and therefore $\frac{\Pi^{\chi}_{H,t+s-1}}{\Pi_{H,t+s}}$ represents the degree of indexation of prices to past inflation; $y_{it+\tau}$ denotes output in period $t+\tau$ for a firm that last reset its price in period t; $y_{t+\tau}$ is the aggregate level of output in time $t+\tau$ and $\varepsilon \geq 1$ is the elasticity of substitution across goods. Let the domestic reset price relative to the domestic price level be $\bar{\Pi}_{H,t} = \frac{\bar{p}_{H,t}}{p_{H,t}}$.

The first order conditions for these intermediate firms are:[14]

$$\frac{k_{t-1}}{l_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t}\frac{p_t}{p_{H,t}}, \tag{B.2.5}$$

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{w_t^{1-\alpha}r_t^{\alpha}}{e^{z_t}}\left(\frac{p_t}{p_{H,t}}\right)^{1-\alpha}, \tag{B.2.6}$$

$$\varepsilon f_t^1 = (\varepsilon - 1) f_t^2, \tag{B.2.7}$$

where

$$f_t^1 = \lambda_t mc_t y_t + \beta\theta E_t\left(\frac{\Pi^{\chi}_{H,t}}{\Pi_{H,t+1}}\right)^{-\varepsilon} f_{t+1}^1, \tag{B.2.8}$$

and

$$f_t^2 = \lambda_t \bar{\Pi}_{H,t} y_t + \beta\theta E_t\left(\frac{\Pi^{\chi}_{H,t}}{\Pi_{H,t+1}}\right)^{1-\varepsilon} f_{t+1}^2\left(\frac{\bar{\Pi}_{H,t}}{\bar{\Pi}_{H,t+1}}\right). \tag{B.2.9}$$

where, following Fernández Villaverde (2010), f_t^1 and f_t^2 are two auxiliary variables.

Taking into account the Calvo's pricing mechanism, the aggregate price index can be expressed as follows:

$$1 = \theta\left(\frac{\Pi^{\chi}_{H,t-1}}{\Pi_{H,t}}\right)^{1-\varepsilon} + (1-\theta)\,\bar{\Pi}_{H,t}^{(1-\varepsilon)}. \tag{B.2.10}$$

B.3 Final goods producers

Final goods producers buy intermediate goods from intermediate goods producers and combine them to obtain the homogeneous final good according to the following Dixit-

[14] Since all intermediate good producers face the same prices and because of market clearing, subscript i can be removed from the previous expression, meaning that all the monopolistically competitive producers choose the same ratio for the production factors they use $\frac{k_{it-1}}{l_{it}}$, so that capital and labor will be expressed in aggregate levels.

Stiglitz technology function:

$$y_t = \left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{B.3.1}$$

where y_t is the aggregate demand of the economy, and $\varepsilon > 1$ is the elasticity of substitution across goods. The final good is sold to households, in the form of private consumption, or to the government, in the form of public consumption, in a perfectly competitive market. These firms maximize profits taking both the price of the intermediate good $p_{H,it}$ and the price of the final good $p_{H,t}$ as given. The domestic price level is given by

$$p_{H,t} = \left(\int_0^1 p_{H,it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}. \tag{B.3.2}$$

B.4 Capital goods producers

These agents operate in a perfectly competitive market and create new capital, x_{t+1}, using investment, i_t, and installed capital, x_t, via the following production function:

$$x_{t+1} = x_t + \left(1 - S\left[\frac{i_t}{i_{t-1}} \right] \right) i_t, \tag{B.4.1}$$

where $S\left[\frac{i_t}{i_{t-1}}\right]$ denotes adjustment costs, such that $S'[\cdot] > 0; S''[\cdot] > 0; S[1] = 0$; and $S'[1] = 0$. Installed capital is previously purchased from entrepreneurs. Let q_t denote the relative price of capital, then discounted profits are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[q_t \left(1 - S\left[\frac{i_t}{i_{t-1}} \right] \right) i_t - i_t \right]. \tag{B.4.2}$$

Market clearing implies that $x_t = (1-\delta) k_{t-1}$, where $\delta \in [0,1]$ is the capital depreciation rate.

The first order condition is the following:

$$q_t \left(1 - S\left[\frac{i_t}{i_{t-1}} \right] - S'\left[\frac{i_t}{i_{t-1}} \right] \frac{i_t}{i_{t-1}} \right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S'\left[\frac{i_{t+1}}{i_t} \right] \left[\frac{i_{t+1}}{i_t} \right]^2 = 1. \tag{B.4.3}$$

The law of motion of capital is given by

$$k_t = (1-\delta) k_{t-1} + \left(1 - S\left[\frac{i_t}{i_{t-1}} \right] \right) i_t. \tag{B.4.4}$$

B.5 Entrepreneurs

Entrepreneurs are in charge of transforming installed capital, x_t, into inputs for use by intermediate goods producers, k_{t-1}. Each period, entrepreneurs buy new capital, k_t, from capital goods producers at a price q_t, to undertake their investment.

Entrepreneurs use both internal and external funds for the purchase of the new installed capital, $q_t k_t$. Internal funds are composed of the end-of-period net worth (or equity of the entrepreneurs), n_t; while external funds consist of loans (or liabilities of the entrepreneurs) borrowed from financial intermediaries, b_t. Therefore the amount they borrow is given by[15]

$$\frac{b_t}{p_t} = q_t k_t \frac{p_{H,t}}{p_t} - n_t. \tag{B.5.1}$$

Their technology is affected by an idiosyncratic shock, ω_{t+1}, which is lognormally distributed with cumulative distribution $F(\omega, \sigma_{\omega,t})$ with parameters $\mu_{\omega,t}$ and $\sigma_{\omega,t}$. I assume that $E_t \omega_{t+1} = 1$ for all t. The dispersion, $\sigma_{\omega,t}$, represents the credit risk of the model[16] and is assumed to follow:[17]

$$\widehat{\sigma}_{\omega,t} = \rho_{\sigma_\omega} \widehat{\sigma}_{\omega,t-1} + \eta_{\sigma_\omega} \varepsilon_{\sigma_\omega,t}, \tag{B.5.2}$$

where $\rho_{\sigma_\omega} \in [0,1]$ is the persistence coefficient, $\varepsilon_{\sigma_\omega,t} \backsim N(0,1)$, and η_{σ_ω} is the volatility of the shock. The shock, $\varepsilon_{\sigma_\omega,t}$, is revealed at the end of the period, just before the investment decisions for $t+1$ are taken.

Let r_{t+1} be the price that the entrepreneur charges to the intermediate good producer per unit of capital rented, and let $q_{t+1}(1-\delta)$ be the cost that the capital good producer assumes for the repurchase of the old non-depreciated capital, paid to the entrepreneur at the end of the period. The ex-post average return of the entrepreneur per unit of investment between t and $t+1$, R^k_{t+1}, can be defined as

$$R^k_{t+1} = \Pi_{H,t+1} \frac{r_{t+1} + q_{t+1}(1-\delta)}{q_t}. \tag{B.5.3}$$

The realization of ω_{t+1} is private information to entrepreneurs, and the contract with financial intermediaries is signed before it is known. This private information leads to a moral hazard problem with costly state verification that is solved via a standard

[15] Notice that this expression means that the contract is set in nominal terms, what implies that the entrepreneurs' networth may be affected by the debt deflation channel.

[16] This credit risk may arise from household overborrowing or from risk-taking in financial markets.

[17] I use the notation $\widehat{x}_t$ to refer to the log-linearized version of variable x_t and $\overline{x}$ for the steady state value of the same variable.

debt contract. As in Bernanke et al. (1999), I consider a costly state verification (CSV) problem: entrepreneurs observe their outcome for free, but financial intermediaries need to pay a cost, proportional to the gross payoff of the entrepreneur's capital.

The standard debt contract specifies a state-contingent non-default repayment, R^l_{t+1}, (dependent on the ex-post realization of R^k_{t+1}) that the entrepreneur promises to pay to the financial intermediary in case of success of the investment project, that is, as long as the return is enough to meet the payment obligations with the financial intermediary. Otherwise the entrepreneur will default.

At the moment of the debt contract agreement there is aggregate uncertainty because R^k_{t+1} is not known yet. Once the entrepreneur has decided on the amount of capital to purchase, $q_t k_t \frac{p_{H,t}}{p_t}$, and therefore the amount of external funds it needs, the entrepreneur and the financial intermediary agree to sign a one period contract given the ex-ante values of $q_t k_t \frac{p_{H,t}}{p_t}$ and $\frac{b_t}{p_t}$. The threshold value of the idiosyncratic shock, ϖ_{t+1}, below which the entrepreneur defaults, is given by

$$R^l_{t+1} b_t = \varpi_{t+1} R^k_{t+1} p_{H,t} q_t k_t. \tag{B.5.4}$$

Summarizing, after the idiosyncratic shock is realized there are two possible scenarios:

- if $\omega_{t+1} > \varpi_{t+1}$ the financial intermediary will get $R^l_{t+1} b_t$ and the entrepreneur will keep the difference between his revenue and the interest payment on the loan, $\omega_{t+1} R^k_{t+1} p_{H,t} q_t k_t - R^l_{t+1} b_t$;

- if $\omega_{t+1} < \varpi_{t+1}$ the entrepreneur defaults and gets nothing while the financial intermediary gets $(1-\mu)\,\omega_{t+1} R^k_{t+1} p_{H,t} q_t k_t$, where $\mu \omega_{t+1} R^k_{t+1} p_{H,t} q_t k_t$ is the cost of monitoring.

If the entrepreneur defaults, it gets nothing. The financial intermediary takes the remaining fraction $(1-\mu)$ of the entrepreneur's return after paying bankruptcy procedures (a fraction μ). Hence, the CSV problem is designed to ensure that whenever the entrepreneur has generated enough revenue to pay its obligations, it has an incentive to do so and to report truthfully. This is what Freixas and Rochet (2008) call the revelation mechanism.

The debt contract also establishes the return R^l_{t+1} the financial intermediary gets from

the entrepreneur, arising from the zero profit condition

$$\left[1-F\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]R_{t+1}^{l}b_{t}+\left(1-\mu\right)\int_{0}^{\varpi_{t+1}}\omega dF\left(\omega,\sigma_{\omega,t}\right)R_{t+1}^{k}p_{H,t}q_{t}k_{t}=s_{t}R_{t}\left(a_{t}+B_{t}\right), \tag{B.5.5}$$

Equation (B.5.5) shows that expected revenues obtained from lending activities must equal the cost of funds the domestic financial intermediary has to pay back to households

Following Fernández-Villaverde (2010), the problem of the entrepreneur is to choose both the leverage ratio and the schedule for ϖ_{t+1} by maximizing its expected net worth

$$\max_{\frac{\frac{b_t}{p_t}}{n_t},\varpi_{t+1}}\frac{R_{t+1}^{k}}{R_{t}}\left[1-\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]\left(1+\frac{\frac{b_t}{p_t}}{n_t}\right), \tag{B.5.6}$$

subject to the zero profit condition of the financial intermediary,

$$\left[\frac{R_{t+1}^{k}}{R_{t}}\left[\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]\left(1+\frac{\frac{a_t+B_t}{p_t}}{n_t}\right)-\frac{\frac{a_t+B_t}{p_t}}{n_t}\right], \tag{B.5.7}$$

and given that in equilibrium $a_t+B_t=b_t$. In the equations above, $F\left(\varpi_{t+1},\sigma_{\omega,t}\right)$ denotes the probability of default and

$$G\left(\varpi_{t+1},\sigma_{\omega,t}\right)=\int_{0}^{\varpi_{t+1}}\omega dF\left(\omega,\sigma_{\omega,t}\right). \tag{B.5.8}$$

Function $\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)$ stands for the share of entrepreneurial earnings accrued to the financial intermediary

$$\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)=\varpi_{t+1}\left[1-F\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]+G\left(\varpi_{t+1},\sigma_{\omega,t}\right). \tag{B.5.9}$$

The first order conditions are given by

$$\begin{aligned}&E_{t}\frac{R_{t+1}^{k}}{R_{t}}\left[1-\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]\\&+\xi_{t}\left\{\frac{R_{t+1}^{k}}{R_{t}}\left[\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]-1\right\}=0,\end{aligned} \tag{B.5.10}$$

and

$$-\Gamma_{\omega}\left(\varpi_{t+1},\sigma_{\omega,t}\right)+\xi_{t}\left[\Gamma_{\omega}\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]=0, \tag{B.5.11}$$

where ξ_t is the Lagrangian multiplier.

After some algebra, I get

$$q_t k_t \frac{p_{H,t}}{p_t} = \left[\frac{\xi_t}{E_t \frac{R^k_{t+1}}{R_t} \left[1 - \Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right]} \right] n_t. \tag{B.5.12}$$

where $q_t k_t \frac{p_{H,t}}{p_t}$ are purchases of capital, as explained before, and where $\frac{R^k_{t+1}}{R_t}$ is the external finance premium, inversely related to the net wealth of the entrepreneur. Everything else equal, a rise in the external finance premium, $efp = \frac{R^k_{t+1}}{R_t}$, that initially reduces the expected probability of default, makes the entrepreneur take on more debt. This generates a decrease in net worth relative to external funds and therefore ends up increasing the expected probability of default.

As mentioned in the description of the households' problem, at the end of every period a fraction γ^e of entrepreneurs survives while the rest die.[18] The net wealth of the exiting entrepreneurs, $\left(1 - \gamma^e\right) n_t$, is paid back to households. The new entrepreneurs replacing exiting ones enter the economy with initial net worth w^e.

The average net wealth (equal to the wealth of the entrepreneur since the leverage ratio is the same for all entrepreneurs) is

$$n_t = \gamma^e \frac{1}{\Pi_t} \left\{ \left[1 - \mu G\left(\varpi_t, \sigma_{\omega,t-1}\right)\right] R^k_t q_{t-1} k_{t-1} \frac{p_{H,t-1}}{p_{t-1}} - s_{t-1} R_{t-1} \frac{b_{t-1}}{p_{t-1}} \right\} + w^e. \tag{B.5.13}$$

B.7 Domestic Financial Intermediaries

Domestic financial intermediaries operate in a perfectly competitive market, receiving deposits from households, a_t, and lending loans to entrepreneurs, b_t. They also make use of the international financial market. In case the demand for loans exceeds the amount of domestic deposits, domestic financial intermediaries obtain funds from the international financial market, $B_t > 0$, that are lent to entrepreneurs in the form of loans. When there is a surplus of domestic deposits relative to the amount of loans that entrepreneurs want to borrow, domestic financial intermediaries deposit the excess of funds in the international financial markets, $B_t < 0$.[19]

[18] Capital demand and capital return by entrepreneurs depend on the evolution of their net worth. And at the same time, entrepreneurs' net worth (equity) depends on their earnings net of interest payments to financial intermediaries. Therefore it is necessary to assume that entrepreneurs have some initial networth, w^e, in order to begin operating.

[19] As markets are incomplete in this model, the international bond is uncontingent, meaning that there is not an outcome for each state of nature. Incomplete markets also imply that the interest rate in one country is not the same as the interest rate of the other country.

Their objective function is given by

$$\left\{\left[1-F\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]R_{t+1}^{l}b_{t}+\left(1-\mu\right)\int_{0}^{\varpi_{t+1}}\omega dF\left(\omega,\sigma_{\omega,t}\right)R_{t+1}^{k}p_{H,t}q_{t}k_{t}-s_{t}R_{t}\left(a_{t}+B_{t}\right)\right\}. \tag{B.7.1}$$

which shows expected returns in case of a successful project, plus revenues in case of default, minus the costs in terms of deposits for the financial intermediary. Variable s_t is a spread that domestic financial intermediaries also pay under the concept of intermediation costs and that is paid back to households in a lump-sum way. Also, following Fernández-Villaverde (2010)

$$s_{t}=1+e^{\overline{s}+\widetilde{s}_{t}}, \tag{B.7.2}$$

and

$$\widetilde{s}_{t}=\rho_{s}\widetilde{s}_{t-1}+\sigma_{s}\varepsilon_{s,t}\text{ where }0<\rho_{s}<1\text{ and }\varepsilon_{s,t}\backsim N(0,1). \tag{B.7.3}$$

Parameter ρ_s is the persistence coefficient and σ_s is the volatility of the shock.

Appendix C

Contract between financial intermediary and entrepreneur

The model includes a productivity shock ω_{t+1} that is lognormally distributed with a cumulative distribution function represented by $F\left(\omega,\sigma_{\omega,t}\right)$, being $\mu_{\omega,t}$ the average and $\sigma_{\omega,t}$ the standard deviation of the distribution where $E_t\omega_{t+1}=1$. From the properties of the lognormal distribution:

$$E_{t}\omega_{t+1}=e^{\mu_{\omega,t}+\frac{1}{2}\sigma_{\omega,t}^{2}}\Rightarrow e^{\mu_{\omega,t}+\frac{1}{2}\sigma_{\omega,t}^{2}}=1\Rightarrow\mu_{\omega,t}+\frac{1}{2}\sigma_{\omega,t}^{2}=0\Rightarrow\mu_{\omega,t}=-\frac{1}{2}\sigma_{\omega,t}^{2}.$$

In the computations to obtain the loglinearized version of the model I use the following equations that are also derived from the properties of the lognormal distribution:

$$\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)=\varpi_{t+1}\left(1-F\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right)+G\left(\varpi_{t+1},\sigma_{\omega,t}\right),$$

$$\Gamma_{\omega}\left(\varpi_{t+1},\sigma_{\omega,t}\right)=1-F\left(\varpi_{t+1},\sigma_{\omega,t}\right),$$

$$G\left(\varpi_{t+1},\sigma_{\omega,t}\right)=1-\phi\left(\frac{\frac{1}{2}\sigma_{\omega,t}^{2}-\log\varpi_{t+1}}{\sigma_{\omega,t}}\right),$$

and

$$G_{\omega}\left(\varpi_{t+1},\sigma_{\omega,t}\right)=\varpi_{t+1}F_{\omega}\left(\varpi_{t+1},\sigma_{\omega,t}\right).$$

Appendix D

Entrepreneur's problem with macroprudential policy

I solve again the problem of the entrepreneur introducing the macroprudential tool in the zero profit condition of the financial intermediary. Therefore:

$$\frac{R_{t+1}^{k}}{R_{t}}\left[\varpi_{t+1}\left[1-F\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]+\left(1-\mu\right)\int_{0}^{\varpi_{t+1}}\omega dF\left(\omega,\sigma_{\omega,t}\right)\right]q_{t}k_{t}\frac{p_{H,t}}{p_{t}}=\eta_{t}\frac{b_{t}}{p_{t}},\quad\text{(D.1)}$$

and taking into account the properties of the lognormal distribution, the zero profit condition of the financial intermediary is:

$$\frac{R_{t+1}^{k}}{R_{t}}\left[\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]q_{t}k_{t}\frac{p_{H,t}}{p_{t}}=\eta_{t}\frac{b_{t}}{p_{t}}.\quad\text{(D.2)}$$

The problem of maximization of the entrepreneur's expected networth requires choosing both the ratio of leverage and the schedule for ϖ_{t+1}:

$$\max_{\frac{\frac{b_{t}}{p_{t}}}{n_{t}},\varpi_{t+1}}\frac{R_{t+1}^{k}}{R_{t}}\left[1-\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]\left(1+\frac{\frac{b_{t}}{p_{t}}}{n_{t}}\right),\quad\text{(D.3)}$$

subject to the zero profit condition of the financial intermediary,

$$\left[\frac{R_{t+1}^{k}}{R_{t}}\left[\Gamma\left(\varpi_{t+1},\sigma_{\omega,t}\right)-\mu G\left(\varpi_{t+1},\sigma_{\omega,t}\right)\right]\left(1+\frac{\frac{b_{t}}{p_{t}}}{n_{t}}\right)-\eta_{t}\frac{\frac{b_{t}}{p_{t}}}{n_{t}}\right].\quad\text{(D.4)}$$

After maximizing the previous expression, the two first order conditions with ξ_t as the Lagrangian coefficient are:

$$E_t \frac{R^k_{t+1}}{R_t} \left[1 - \Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] + \xi_t \left\{ \frac{R^k_{t+1}}{R_t} \left[\Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu G\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] - \eta_t \right\} = 0, \quad \text{(D.5)}$$

and

$$-\Gamma_\omega\left(\varpi_{t+1}, \sigma_{\omega,t}\right) + \xi_t \left[\Gamma_\omega\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu G\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] = 0. \quad \text{(D.6)}$$

From this last first order condition I write the Lagrangian as:

$$\xi_t = \frac{\Gamma_\omega\left(\varpi_{t+1}, \sigma_{\omega,t}\right)}{\Gamma_\omega\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu G\left(\varpi_{t+1}, \sigma_{\omega,t}\right)} = \frac{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)}{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu \varpi_{t+1} F_\omega\left(\varpi_{t+1}, \sigma_{\omega,t}\right)}, \quad \text{(D.7)}$$

and then rewriting:

$$E_t \frac{R^k_{t+1}}{R_t} \left[1 - \Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] = E_t \left[\frac{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)}{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu \varpi_{t+1} F_\omega\left(\varpi_{t+1}, \sigma_{\omega,t}\right)}\right] \left\{ \eta_t - \frac{R^k_{t+1}}{R_t} \left[\Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu G\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] \right\}, \quad \text{(D.8)}$$

what combined with the zero profit condition of the financial intermediary gives:

$$E_t \frac{R^k_{t+1}}{R_t} \left[1 - \Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right] = E_t \left[\frac{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right)}{1 - F\left(\varpi_{t+1}, \sigma_{\omega,t}\right) - \mu \varpi_{t+1} F_\omega\left(\varpi_{t+1}, \sigma_{\omega,t}\right)}\right] \eta_t \frac{n_t}{q_t k_t \frac{p_{H,t}}{p_t}}, \quad \text{(D.9)}$$

also written as:

$$q_t k_t \frac{p_{H,t}}{p_t} = \left[\frac{\xi_t \eta_t}{E_t \frac{R^k_{t+1}}{R_t} \left[1 - \Gamma\left(\varpi_{t+1}, \sigma_{\omega,t}\right)\right]}\right] n_t. \quad \text{(D.10)}$$

Finally, the average net wealth of the entrepreneur, taking into account the macroprudential instrument, becomes:

$$n_t = \gamma^e \frac{1}{\Pi_t} \left\{ \left[1 - \mu G\left(\varpi_t, \sigma_{\omega,t-1}\right)\right] R^k_t q_{t-1} k_{t-1} \frac{p_{H,t-1}}{p_{t-1}} - R_{t-1} \frac{b_{t-1}}{p_{t-1}} \eta_t \right\} + w^e. \quad \text{(D.11)}$$

Appendix E. Alternative analysis for non-coordinated macroprudential policy

Figures 5 and 6 compare the no macroprudential and the country-targeted macroprudential cases with two alternative scenarios. Both consist of the implementation of a country-targeted toolkit but in a non-coordinated way, that is, only one of the countries implement macroprudential policy: the Home country macroprudential scenario is represented by the dotted line and the Foreign country macroprudential scenario by the dash-dotted line. The dashed line represents the country-targeted macroprudential scenario in which both countries implement macroprudential measures and the solid line the no macroprudential scenario.

E.1 Macroprudential policy at the Home country

The home country is not affected by what the foreign country does, that is it does not care about coordination. Therefore, for the home country this non-coordinated scenario is equivalent to the case in which both countries implement macroprudential policy. The foreign country however attains more stability when the home country is the only one that applies macroprudential policy. In this case the foreign country is even more stabilized than when both countries undertake macroprudential measures. This situation can lead the foreign country to free-ride.

E.2 Macroprudential policy at the Foreign country

As mentioned previously, the home country is not affected by what the foreign country does, so for the home country this non-coordinated scenario is equivalent to the no macroprudential policy case. By constrast, the foreign country attains more stability than in the no macroprudential scenario but less stability than in any other case.

Tables

Table 2: Calibration of the parameters and steady states

Parameter	**Description**	**Value**	**Source**
β	Discount factor	0.999	Fernández-Villaverde (2010b)
h	Consumption habits	0.5	Fernández-Villaverde (2010b)
n	Size of the home country	0.5	Faia (2001)
$\frac{c_F}{y}$	Imports from the foreign country-to-GDP	0.1	Own calibration to obtain a ratio $\frac{\bar{B}}{y} = 1.88$
$\frac{c_H^*}{y^*}$	Exports to the foreign country-to-GDP	0.11	Own calibration to obtain a ratio $\frac{\bar{B}}{y} = 1.88$
ζ	Substitutability between domestic and foreign goods	1.5	Faia (2001)
Ω	Debt elasticity of the country premium	0.0043	Quint and Rabanal (2014)
t	Steady state value for the terms of trade	1	Faia (2001)
ϑ	Frisch elasticity of labor	0.5	Fernández-Villaverde (2010b)
α	Capital share of the intermediate production function	0.33	Fernández-Villaverde (2012)
δ	Capital depreciation rate	0.023	Fernández-Villaverde (2012)
θ	Calvo pricing parameter	0.8	Fernández-Villaverde (2010b)
ε	Elasticity of substitution across goods	8.577	Fernández-Villaverde (2012)
χ	Degree of indexation	0.6	Fernández-Villaverde (2010b)
$pdef$	Annual probability of default	0.03	Bernanke et al. (1999)

Parameter	Description	Value	Source
μ	Bankruptcy costs	0.15	Fernández-Villaverde (2012)
$s = s^*$	Average spread	1.0025	Fernández-Villaverde (2012)
$\bar{\gamma}^e = \bar{\gamma}^{e*}$	Entrepreneurs exit coefficient	3.67	Fernández-Villaverde (2010b)
$\tau_l = \tau_l^*$	Steady state of labor income tax rate	0.24	Fernández-Villaverde (2010b)
$\tau_r = \tau_r^*$	Steady state of capital income tax rate	0.42	Own calibration to obtain a ratio $\frac{\bar{B}}{y}$ of 1.88
$\Pi = \Pi^* = \Pi_H = \Pi_F$	Target gross inflation	1.005	Fernández-Villaverde (2010b)
$l = l^*$	Time devoted to work	1/3	Fernández-Villaverde (2010b)
$q = q^*$	Tobin's q. Price of capital	1	Fernández-Villaverde (2010b)
R^d	Steady state of interest rate on home public debt	$\frac{\Pi}{\beta}$	Fernández-Villaverde (2010b)
R	Steady state of interest rate on home deposits	$\frac{R^d-1}{1-\tau_R}+1$	Fernández-Villaverde (2010b)
R^{d*}	Steady state of interest rate on foreign public debt	$\frac{\Pi^*}{\beta}$	Fernández-Villaverde (2010b)
R^*	Steady state of interest rate on foreign deposits	$\frac{R^{d*}-1}{1-\tau_R^*}+1$	Fernández-Villaverde (2010b)
$\frac{\bar{b}}{k} = \frac{\bar{b}^*}{k^*}$	Loan-to-capital ratio	1/3	Fernández-Villaverde (2010b)
$\frac{g}{y} = \frac{g^*}{y^*}$	Government expenditure-to-GDP ratio	0.2	Gomes and Seoane (2018)
$\frac{d}{y} = \frac{d^*}{y^*}$	Public debt-to-GDP ratio	0.6	Gomes and Seoane (2018)
$S^{"}$ [1]	Capital adjustment costs	14.477	Fernández-Villaverde (2012)
ρ_ϕ	Persistence of preference shock	0.95	Fernández-Villaverde (2012)
σ_ϕ	Volatility of preference shock	0.156	Own calibration

Parameter	Description	Value	Source
ρ_s	Persistence of spread shock	0.95	Fernández-Villaverde (2012)
σ_s	Volatility of spread shock	4.140	Own calibration
γ_g	Persistence parameter of government spending shock	0.95	Fernández-Villaverde (2012)
σ_g	Volatility of government spending shock	0.007	Gomes and Seoane (2018)
ρ_z	Persistence of technology shock	0.95	Fernández-Villaverde (2012)
σ_z	Volatility of technology shock	0.0341	Own calibration
ρ_σ	Persistence of credit risk shock	0.95	Fernández-Villaverde (2012)
η_σ	Volatility of credit risk shock	0.560	Gomes and Seoane (2018)
γ_R	Persistence of monetary policy shock	0.95	Fernández-Villaverde (2012)
σ_m	Volatility of monetary policy shock	0.003	Gomes and Seoane (2018)
$\gamma_\Pi (1-\gamma_R)$	Response of intervention rate to changes in inflation	1.5	Fernández-Villaverde (2012)
d_g	Response of government spending to changes in public debt	-0.01	Own calibration
d_g^*	Response of foreign government spending to changes in foreign public debt	-0.01	Own calibration
$\gamma_\eta = \gamma_\eta^*$	Response of macroprudential tool to changes in credit market conditions	0 or 1.75	Own calibration
$\eta = \eta^*$	Steady state value of macroprudential instrument	1	Quint and Rabanal (2014)

Table 3: Main steady state values

Description	Home country	Foreign country
Imports-to-GDP ratio, $\frac{c_F}{y}$ or $\frac{c_H^*}{y^*}$	0.10	0.11
Fraction of consumption produced in the other country, φ	0.17	0.18
International debt-to-GDP ratio, $\frac{\bar{B}}{y}$	1.88	-1.88
Private consumption-to-GDP ratio, $\frac{c}{y}$	0.60	0.62
Private investment-to-GDP ratio, $\frac{i}{y}$	0.18	0.18
Public spending-to-GDP ratio, $\frac{g}{y}$	0.2	0.2
Tax rate on capital, τ_r	0.42	0.42
Tax rate on labor, τ_l	0.24	0.24
Tax rate on consumption, τ_c	0.09	0.06

Note: These values confirm that the home country is a net borrower and the foreign country a net lender in steady state. Moreover, the home country is a net exporter while the foreign country is the net importer.

Table 4: Standard deviations for alternative macroprudential implementations under a credit risk shock.

Variable	No macroprudential tool	Country-targeted macroprudential	Supranational macroprudential
Home country			
Output	0.0174	0.0140	0.0090
Private debt	0.0806	0.0457	0.0666
Public debt	0.0424	0.0298	0.0203
Foreign country			
Output$*$	0.0013	0.0011	0.0129
Private debt$*$	0.0077	0.0032	0.0213
Public debt$*$	0.0393	0.0153	0.0407

Note: These results are the standard deviations to a standard deviation credit risk shock with $\eta_\sigma = 0.560$

Table 5: Correlation between public and private debt for alternative macroprudential implementations under a credit risk shock.

Variable	No macroprudential tool	Country-targeted macroprudential	Supranational macroprudential
Home country	-0.8135	0.7757	-0.7262
Foreign country	-0.3539	0.9499	-0.8928

Note: These results are the correlations to a standard deviation credit risk shock with $\eta_\sigma = 0.560$

Table 6: Robustness analysis. Second order moments in the event of a spread shock.

Variable	No macroprudential tool	Country-targeted macroprudential	Supranational macroprudential
Home country			
$\sigma(Output)$	0.0174	0.0082	0.0131
$\sigma(Privatedebt)$	0.0218	0.0106	0.0164
$\sigma(Publicdebt)$	0.0417	0.0204	0.0356
$\rho(\frac{B}{Y}, \frac{D}{Y})$	-0.4500	-0.8473	-0.7261
Foreign country			
$\sigma(Output*)$	0.0012	0.0013	0.0042
$\sigma(Privatedebt*)$	0.0066	0.0038	0.0112
$\sigma(Publicdebt*)$	0.0346	0.0162	0.0270
$\rho(\frac{B*}{Y*}, \frac{D*}{Y*})$	-0.1165	0.8991	0.4151

Note: These results are the second order moments to a standard deviation spread shock with $\sigma_s = 4.140$.

Table 7: Robustness analysis. Second order moments in the event of a preference shock.

Variable	No macroprudential tool	Country-targeted macroprudential	Supranational macroprudential
Home country			
$\sigma(Output)$	0.0174	0.0146	0.0150
$\sigma(Privatedebt)$	0.0513	0.0440	0.0470
$\sigma(Publicdebt)$	0.0799	0.0917	0.0839
$\rho(\frac{B}{Y}, \frac{D}{Y})$	0.9252	0.9888	0.9502
Foreign country			
$\sigma(Output*)$	0.0054	0.0081	0.0105
$\sigma(Privatedebt*)$	0.0078	0.0088	0.0071
$\sigma(Publicdebt*)$	0.0699	0.0856	0.0931
$\rho(\frac{B*}{Y*}, \frac{D*}{Y*})$	-0.1769	0.9637	0.7777

Note: These results are the second order moments to a standard deviation preference shock with $\sigma_\phi = 0.156$.

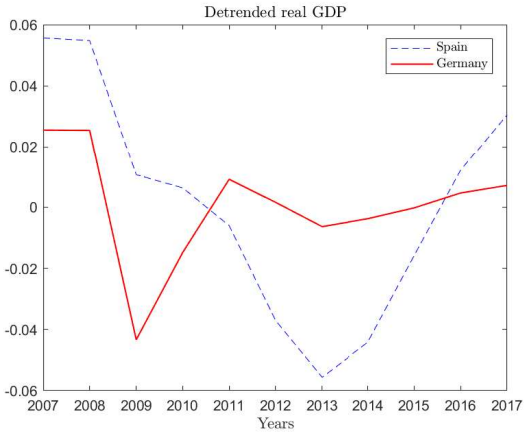
Table 8: Robustness analysis. Second order moments in the event of a technology shock.

Variable	No macroprudential tool	Country-targeted macroprudential	Supranational macroprudential
Home country			
$\sigma(Output)$	0.0173	0.0137	0.0150
$\sigma(Privatedebt)$	0.0083	0.0089	0.0091
$\sigma(Publicdebt)$	0.0285	0.0257	0.0266
$\rho(\frac{B}{Y}, \frac{D}{Y})$	-0.1673	0.0025	-0.2326
Foreign country			
$\sigma(Output*)$	0.0014	0.0020	0.0029
$\sigma(Privatedebt*)$	0.0060	0.0051	0.0054
$\sigma(Publicdebt*)$	0.0282	0.0332	0.0351
$\rho(\frac{B*}{Y*}, \frac{D*}{Y*})$	0.2137	0.8922	0.7305

Note: These results are the second order moments to a standard deviation technology shock with $\sigma_z = 0.0341$.

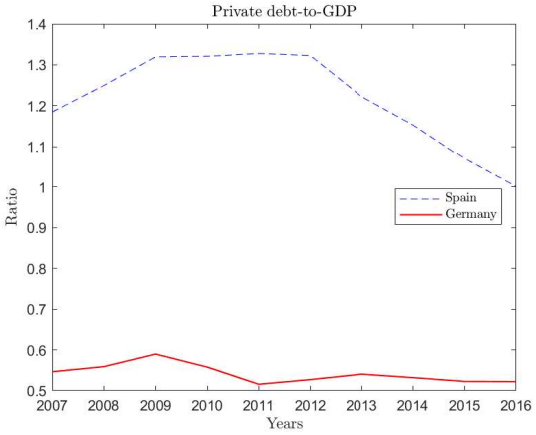
Figures

Figure 1: GDP recovery paths in Germany and Spain for period 2007-2017



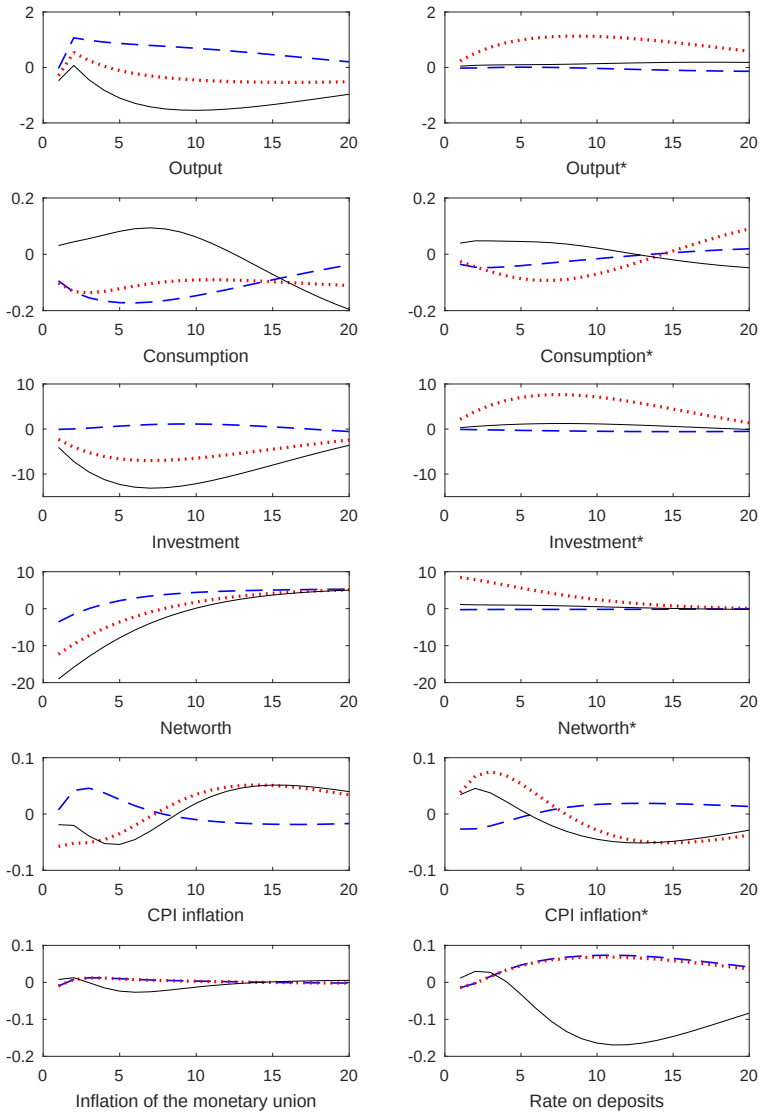
Note: The series plotted in this graph is the real GDP series detrended through the Hodrick Prescott filter.

Figure 2: Private debt in Germany and Spain for period 2007-2017



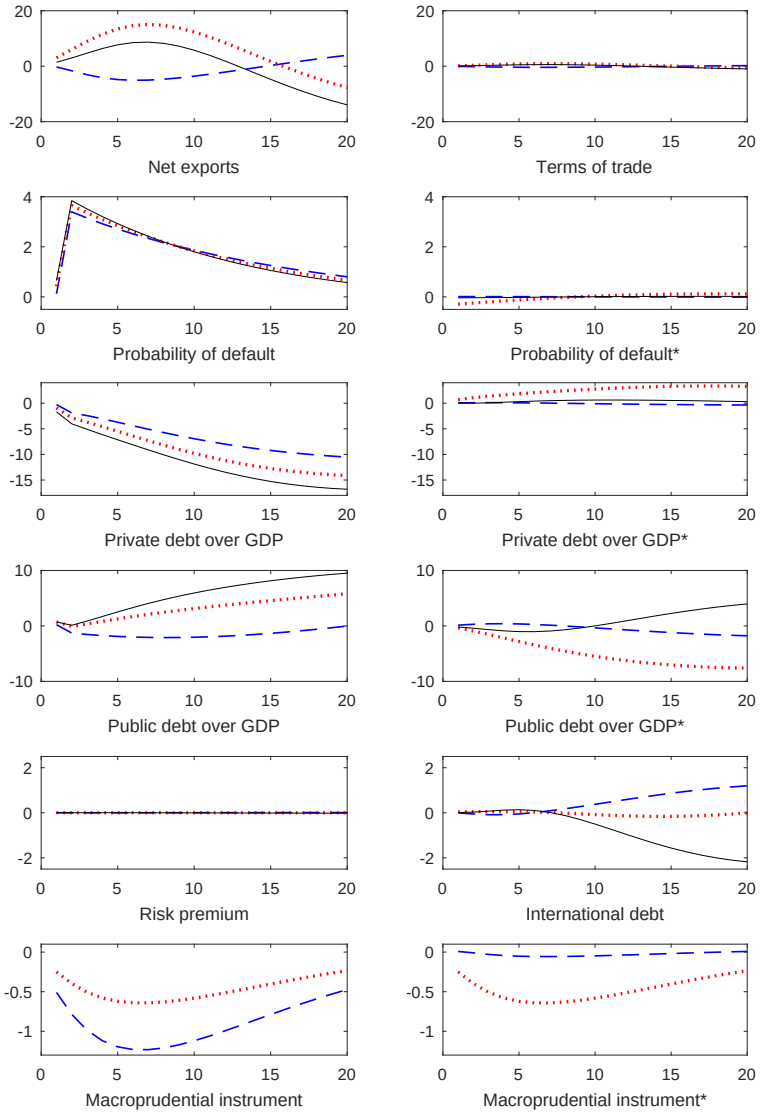
Note: The series plotted in this graph is the real credit to the private non financial sector-to-real GDP ratio.

Figure 3: Impulse response functions to a 1 standard deviation rise in credit risk.



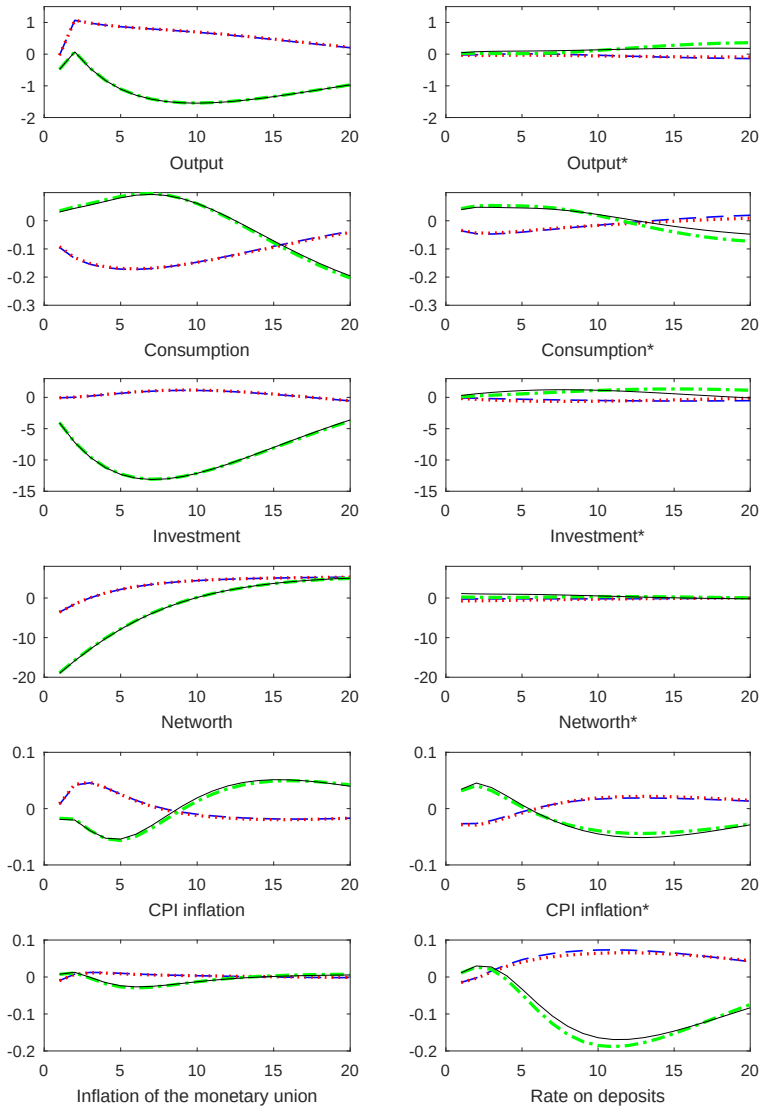
Note: The dashed line represents the country-targeted macroprudential scenario, the dotted line the supranational macroprudential scenario and the solid line the no macroprudential scenario. Variables are expressed in percentage points of deviations from steady state. Except for the last row, home variables are in the left column and foreign variables in the right column.

Figure 4: Impulse response functions to a 1 standard deviation rise in credit risk.



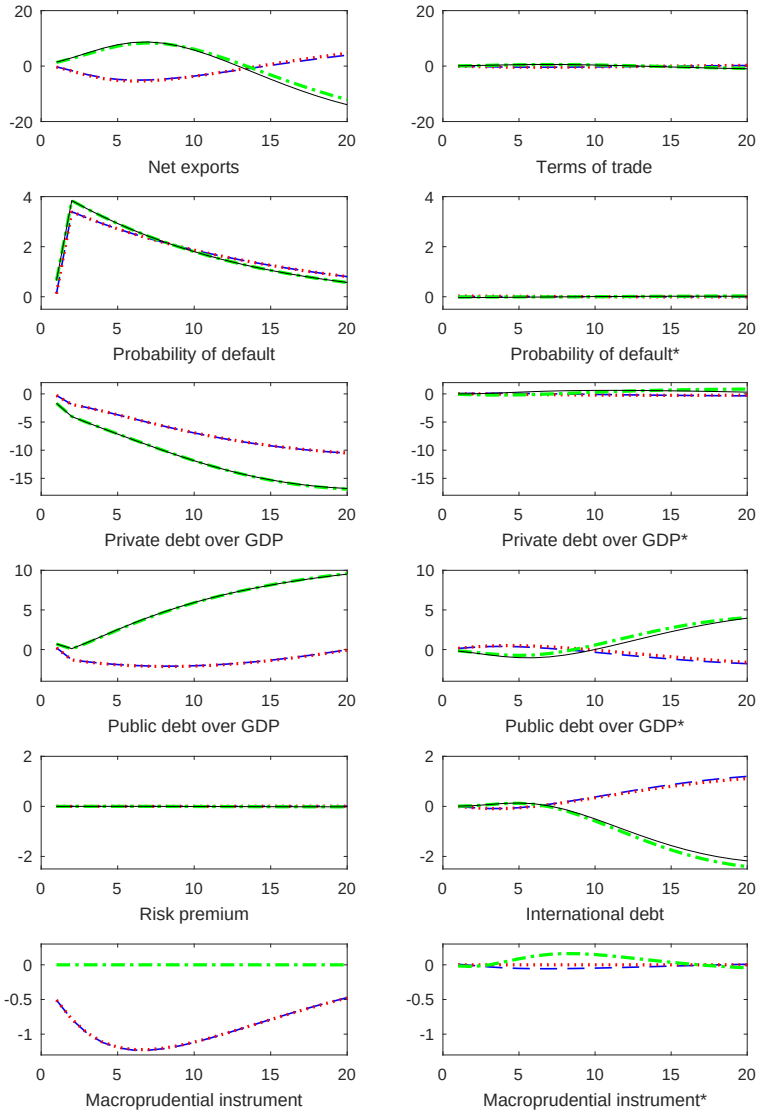
Note: The dashed line represents the country-targeted macroprudential scenario, the dotted line the supranational macroprudential scenario and the solid line the no macroprudential scenario. Variables are expressed in percentage points of deviations from steady state. Home variables are in the left column and foreign variables in the right column in the second, third and fifth rows.

Figure 5: Impulse response functions to a 1 standard deviation rise in credit risk.



Note: The dashed line represents the country-targeted macroprudential scenario in which both countries apply the instrument, the dotted line the Home country macroprudential scenario, the dash-dotted line the Foreign country macroprudential scenario and the solid line the no macroprudential scenario. Variables are expressed in percentage points of deviations from steady state. Except for the last row, home variables are in the left column and foreign variables in the right column.

Figure 6: Impulse response functions to a 1 standard deviation rise in credit risk.



Note: The dashed line represents the country-targeted macroprudential scenario in which both countries apply the instrument, the dotted line the Home country macroprudential scenario, the dash-dotted line the Foreign country macroprudential scenario and the solid line the no macroprudential scenario. Variables are expressed in percentage points of deviations from steady state. Home variables are in the left column and foreign variables in the right column in the second, third and fifth rows.