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Fractional Integration Analysis of Rare Earth Material Price Dynamics for Mechanical System Design and Sustainable Manufacturing

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ABSTRACT

Rare earth elements (REEs) are strategic resources essential for defense mechanical systems, digital industries, and clean energy technologies. This study examines the persistence and long-term dynamics of REE prices from September 2012 to May 2022 using the ARFIMA mechanical model, which captures both short- and long-memory behavior. Additionally, descriptive statistics and fractional integration analysis are employed to evaluate distributional characteristics and interrelationships among REE prices. The descriptive findings reveal notable heterogeneity across elements, with neodymium, praseodymium, terbium, and dysprosium exhibiting higher mean values, greater volatility, and positive skewness, indicating higher sensitivity to market shocks. The ARFIMA results show that most REE prices are fractionally integrated and mean-reverting, suggesting that shocks are largely transitory. However, neodymium, praseodymium, terbium, and dysprosium display high persistence and near unit-root behavior, implying that shocks may have permanent effects. These findings highlight the importance of critical REEs for supply chain resilience, risk management, and sustainable energy transition policies.

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1. Introduction

The need for different types of essential raw materials to fuel the world towards renewable energy resources has been on the rise. Of these essential raw materials, REEs have emerged as an essential part of the latest technology in defense mechanical systems, electric cars, wind turbines, and digital technology. REEs have thus emerged at the center of economic, environmental, and geopolitical issues due to their economic criticality, especially with regard to energy security and climate change mitigation.

While REEs are not scarce in nature, they have emerged as an essential part of economic criticality due to the supply chain. REEs are environmentally intensive, technologically complex, and geographically concentrated. China has dominated the REE supply chain globally, with 97% of the world's production being sourced from China. Abo-Khalil (Abo-Khalil, 2024), have shown that Haque et al. (Haque et al., 2014) have shown that China dominates the REE supply chain with regard to export restrictions, regulatory tightening, and disruptions due to the COVID-19 pandemic.

Apart from these geopolitical aspects, there are special chemical and physical properties of REEs that restrict their substitutability and hence increase their importance. The special electronic configuration of REEs enables stable complexes and is responsible for their use in permanent magnets, catalysts, and efficient applications (Wood, 1990). These special properties make some REEs, such as dysprosium, terbium, and neodymium, essential for renewable energy applications and electric vehicles, thereby establishing a direct link between REEs and the energy transition process.

From an economic perspective, one of the important aspects, though not properly explored, is related to the dynamics of REE prices. In fact, it is important to determine whether price changes have permanent or transitory effects. This is important in view of the potential persistence of price dynamics, which may influence their long-run impact on supply shocks and hence limit development in general and renewable energy applications in particular. Empirical results suggest a strong association between REE prices and renewable energy consumption as well as macroeconomic variables (Apergis & Apergis, 2017).

In addition to the increased importance of the global economy and technology, the financialization of commodity markets has had a further impact on the price patterns and trends of REE. This has been in the form of increased co-movements in the price movements of REE and other strategic commodities and financial assets (Dziubaniuk et al., 2023; Jowitt, 2022). The policy regime, as indicated by the subsidies and trade policies in green energy, trade, and resource nationalism, continues to have a significant impact on the supply side in the REE sector. Hence, the price patterns and trends in the REE sector would also be related to the understanding of the financial market and the structural policy regime (Ferreira & Critelli, 2022).

Moreover, the increased volatility and uncertainty in the commodity markets worldwide have also increased the importance of using advanced tools to analyze the price dynamics of REEs. Indeed, the conventional methods based on time series mechanical models might not be able to address the complex memory effects and persistence in the price dynamics of REEs (Ding et al., 1993). In such contexts, the application of fractional integration methods might be more effective in the price dynamics of REEs. This is because the methods might allow for non-integer values in the order of differencing, thus providing more insights to understand the transience or otherwise of the price dynamics in the REE industry. Such information would be useful to policymakers and stakeholders in the industry because it would enable the adoption of effective tools to manage the supply chain and technology development in the renewable energy industry (Deng et al., 2021; Malhotra et al., 2023).

1.1 Research Objective

The association between the mechanical system design and the economic system is significantly influenced by the research and cost of rare earth materials. This enables the engineers to make the most important decisions regarding the mechanical system design. This method of design of the mechanical system enables the building of a reliable and dynamic system and a sustainable manufacturing process.

1.2 Key Contributions

- Focuses on the optimization of design for the mechanical system with consideration of material cost variability and performance.
- Develops a framework for analyzing the price dynamics of rare earth materials for decision-making in engineering.
- Takes into consideration the long-run price behavior using fractional integration.
- Introduces economic factors for better cost-effective design.
- Supports the concept of sustainable manufacturing with efficient material use.

The importance of fractional integration and long-memory processes in describing commodity prices has been emphasized more and more in the literature on commodity pricing. The use of the fractional integration mechanical model has the advantage of capturing intermediate levels of persistence in commodity prices, which cannot be easily accounted for by the conventional ARIMA mechanical model. The mechanical model has been applied to the rare earth markets with satisfactory results. While Claudio-Quiroga et al. (Claudio-Quiroga et al., 2023) examine mineral price persistence in relation to the development of China's electric vehicle industry. Other contributions focus on co-movements, common features, and spillover mechanisms across commodity markets (Issler et al., 2014; Wei et al., 2023)dee.

According to research on the global financial crisis, China's manufacturing output was negatively impacted by the U.S. subprime mortgage crisis. The results show that the impacts were rapid and substantial, resulting in a steady drop until plateauing. The study examined data from 2005 to 2008 and used an ARIMA Intervention mechanical model. The study demonstrates how such effects, although temporary, are critical and how intervention mechanical models are effective in measuring and capturing economic shocks from outside forces (Chung et al., 2009). The predictive performance of leading economic indicators has been reassessed using a real-time framework that relies on historically available provisional data and recursive out-of-sample forecasts. The findings indicate a notable deterioration in forecasting accuracy compared to traditional approaches, emphasizing the challenges and limitations of fusing such indicators in dynamic and continuously revised economic environments (Diebold & Rudebusch, 1991).

Research into a single-molecule rotary switch aims to understand how electrical energy can be converted into mechanical motion. Atomic force microscopy and electron tunneling are used in the research to show that the switch has two stable states with stochastic switching between them. Although the potential efficiency is high with a single switch, the energy conversion is quite low, which is an issue with the energy transmission process, where elastic tunneling is the major factor (Larson et al., 2020). Export restrictions on metals and minerals have been quite common across the world. These have been implemented to ensure domestic security, to conserve resources, and to promote development. This research aims to explore the export restrictions imposed on minerals, specifically REEs, where China is the major player in the world market. The research findings have highlighted the export restrictions, their importance in the world market where the resources are scarce, and how they affect other countries, especially the Western world (Mancheri, 2016).

Earlier research works have focused on the environmental hazards associated with REEs by defining thresholds such as the Maximum Permissible Concentration (MPC) and Negligible Concentration (NC). However, due to the lack of toxicity information, especially in aquatic media, high factors have been used. Research has indicated that the MPCs in freshwater media are higher than those in saltwater media. Additionally, environmental concentrations in the media sometimes surpass the NC levels (Zheng et al., 2022).

Though these studies offer valuable insights, an important question remains unanswered. Do individual REEs display heterogeneous long memory properties? Are strategically important REEs distinguished by greater price persistence than less scarce rare earth elements? These studies are based on aggregate price indices, broad categories of minerals, and spillover effects based on multiple variate approaches. These approaches are inherently unable to capture heterogeneity in long memory properties at the individual element level.

In this regard, the analysis goal is to use fractional integration techniques to objectively investigate the long memory characteristics of REE pricing. The analysis uses weekly price series for major REEs over the period 2012-2022. At the individual REE level, the study distinguishes between mean-reverting qualities and strong long memory properties or near unit root properties using the Autoregressive Fractionally Integrated Moving Average (ARFIMA) mechanical model. This will enable us to gain insights into the response of REE prices to external shocks.

The application of these approaches in the markets for minerals and REEs is still at an early stage. Prior literature includes analyses of REE price behavior based on ARIMA-type approaches (García et al., 2018) or comparing different approaches to mechanical modeling volatility and persistence. Meanwhile, prior studies provide limited research and explored heterogeneity of long-memory behavior among individual REEs. In addition, there remains uncertainty about whether strategically important REEs exhibit persistence patterns that differ from those of more commonly available elements, which has important consequences for supply stability, resilience planning, and risk assessment. Building on recent research examining aggregate indices, it first provides a comparative analysis of long-memory behavior among individual REEs based on a comparable weekly data set. Second, it identifies a grouping of strategically important REEs, including praseodymium, terbium, neodymium, and dysprosium, whose prices exhibit considerable persistence and possibly permanent responses to shocks, and it shows significant variability in price persistence among individual REEs. Third, it talks about how the energy revolution will affect forecasting, risk management, and supply chain resilience.

1.3 Research Gaps

The current literature on dynamics of prices for commodities and REEs is based on standard ARIMA techniques, use of aggregate indices, or use of multiple variables where co movements and spillovers play the central role. The primary limitation of this method is the reliance on whole-number integer orders of integration while intermediate states and differences in persistence properties of different elements are ignored. At the same time, the existing literature emphasizes shocks in the economy, environmental problems, and export controls but ignores the element-specific long-memory properties.

This research fills in this gap in the literature by using ARFIMA methodology to analyze the dynamics of individual REEs prices. This is how the remainder of the research is structured. The data and price indexes utilized are shown in Section 2. The methodological framework based on Autoregressive Fractionally Integrated Moving Average modeling and fractional integration is presented in Section 3. Section 4 outlines and interprets the results obtained in the study. Section 5 discuss the main conclusions and provides recommendations for future research directions.

2. Data

In this analysis, the use of the weekly indices of the key REEs from June 2012 to May 2022 results in 519 observations for the series. The Thomson Reuters Eikon database was used as the source of the data. The database offers standardized and popularly used indices in the commodities markets.

The REEs selected for this analysis include neodymium, lanthanum, praseodymium, samarium, europium, cerium, gadolinium, terbium, erbium, dysprosium, and yttrium. These REEs vary from relatively abundant to strategically critical in clean energy technologies.

All the series in this analysis were examined in their original level form. Fractional integration techniques allow the series to be examined without the need for differencing.

All series in this study are analyzed in their original level form. Fractional integration techniques allow for assessing persistence without differencing.

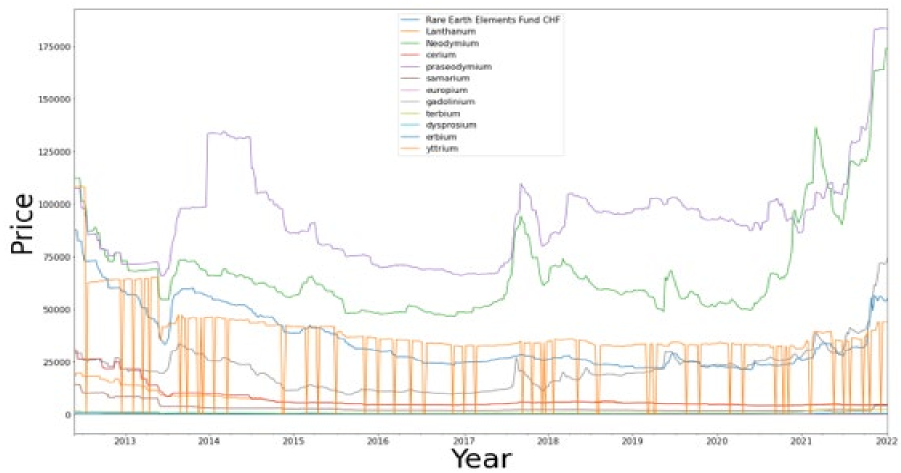

Figure 1: Price Indexes of Individual REEs

Figure 1 below shows a graphical representation of the evolution of individual REEs' price indices for our sample period. While there are some series with stable price developments over long periods, there are episodes characterized by high volatility and price movements, especially during significant market disruptions. However, as seen in the periods after China's export restriction policies in the early 2010s and those due to disruptions from the COVID-19 pandemic, there are significant price movements for some REEs, especially for. This supports our argument for the potential long-run effects of these price shocks and hence supports our argument for price persistence analysis.

3. Materials and Methods

The analysis evaluates the behavior of data and relationships based on the analysis of weekly REE prices (2012–2022) with the help of descriptive statistics and Fractional Integration analysis. Stationarity is tested using the ADF and PP unit root tests. The proposed approach captures both short-run fluctuations and long-run persistence in prices through the ARFIMA framework, while the most appropriate model specification is determined using the Akaike (Akaike, 1979) Information Criterion (AIC) and Bayesian Information Criterion (BIC) to achieve the best fit.

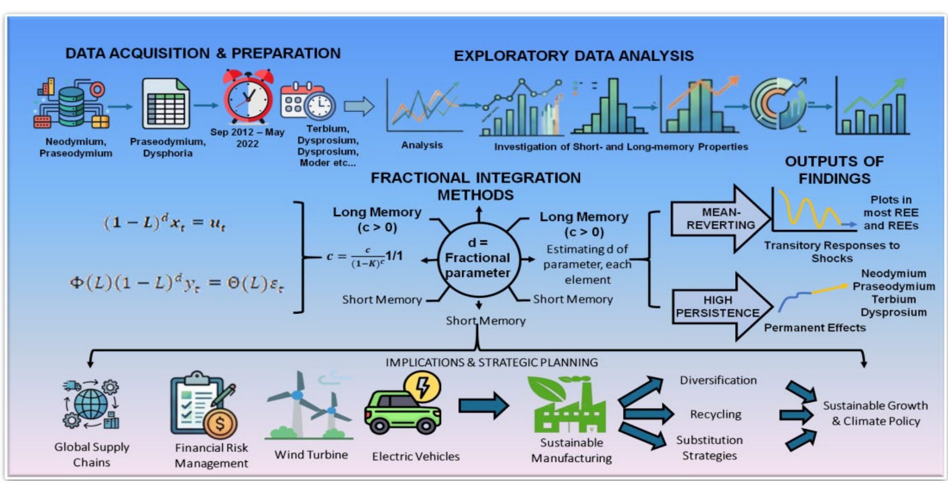

Figure 2: Methodology Flow for REE price analysis using ARFIMA

Figure 2 depicts the complete process, from data preparation and collection to statistical testing. It offers more illustrations of ARFIMA modeling and the evaluation of long-memory behavior in rare earth element prices.

3.1 Statistical Analysis

This analysis explores the price movement of REE using correlation, descriptive statistics, unit root tests, and ARFIMA mechanical models. The descriptive statistics are based on the data properties. ARFIMA mechanical models are based on long memory, whereas unit root tests are based on initial stationarity. IBM SPSS Statistics 26 is the programming tool for all basic statistical testing.

3.2 Descriptive Statistics

The key features of REE price data, such as central tendency and fluctuation, are summarized using descriptive statistics. They offer a comprehensive comprehension of the distribution of prices, volatility, and general data behavior.

$$\sigma = \sqrt{\frac{1}{m-1}\sum_{i-1}^{m}\left(Y_i - \underline{Y}\right)^2} \tag{1}$$

Here, Y_i reflects both the overall number of observations and individual REE price observations, $\underline{Y}$ denotes the average price, and σ indicates the dispersion (variability) of price around the mean shown in Equation (1).

3.2.1 ARFIMA (p, d, q) Model

Economic variables have been classified based on using conventional time-series mechanical models and common unit root tests as either stationary I(0) or non-stationary I (1). However, a growing body of research shows that all core data process demonstrates partial integration or persistent long-term dependence, the setests are weak (Diebold & Rudebusch, 1991; Hassler & Wolters, 1994). In such situations, only integral uses of the Integration order would be used, which would give erroneous Results regarding mean reversion and persistence.

Because fractional integration permits the differencing parameter to take non-integer values, it offers a more flexible framework. A time series y_t was fractionally integrated of order d, denoted $I(d)$, if it satisfies:

$$(1-L)^d x_t = u_t, \tag{2}$$

The term $(1-L)$ is a form of fractional integration applied to research persistence in the price series of rare earth elements to make sustainable manufacturing decisions shown in Equation (1). In this case, u_t represents the price of REE at time$t$, L is the long-range dependence is measured by the fractional differencing coefficient. and d is the lag operator that takes past pricing into account. A non-moving error process is represented by x_t. The d value is used to identify the temporary or permanent price shocks, which is essential in the design of mechanical systems and cost stability. To capture both short-term fluctuations and long-term persistence, this study applied the ARFIMA model. The specification of the ARFIMA (p,d,q) model is presented in Equation (2).

To examine short-term variations alongside long-term persistence, this study employs the ARFIMA mechanical model. The ARFIMA (p, d, q) specification can be written as:

$$\Phi(L)(1-L)^d y_t = \Theta(L)\varepsilon_t, \tag{3}$$

Equation (3) shows ARFIMA model for rare earth material price dynamics, where y_t denotes the price series influencing material cost in mechanical system design and d captures long-memory persistence affecting long-term cost stability. $\Phi(L)$ is the autoregressive component reflecting past price effects, while $\Theta(L)$ represents short-term shocks such as supply disruptions impacting manufacturing. The lag operator L indicates time dependence in prices, and ε_t is the random error capturing unexpected market fluctuations. This model assists in evaluating cost uncertainty and sustainability risks in engineering.

To obtain precise and dependable estimates for the fractionally integrated framework, the differencing coefficient d together with the autoregressive and moving average components were calculated through the exact maximum likelihood approach described by Sowell (Sowell, 1992).

Alternative mechanical models with varying values for the autoregressive and moving-average terms p and q, ranging from 0 to 2, are used to pick the mechanical model. It is a reasonable compromise between the complexity and simplicity of the mechanical models that are presented here, based on the weekly frequency and the simple size. The optimal mechanical model for each time series is then chosen using the BIC (Pawar & Ewing, 2022) and the AIC Fan et al. (Fan et al., 2023), which are intended to optimize mechanical model fit and reduce over-parameterization.

4. Empirical Results

The empirical analysis is done based on a stepwise process of applying descriptive statistics, unit root testing, and fractional integration techniques (ARFIMA). From the descriptive statistics, it can be seen that the price levels of rare earth elements are highly volatile and do not conform to a normal distribution. From the unit root testing using ADF and PP, there is inconclusive evidence of stationarity, which suggests that the conventional methods have their flaws. The ARFIMA results show that there is heterogeneity in the series, where some parts are mean-reverting ($d < 1$), while others are non-stationary ($d \geq 1$).

4.1 Descriptive Statistics

The descriptive statistics include quantitative techniques that describe the main characteristics of data, including measures of central tendency, dispersion, standard deviation (SD), skewness, and kurtosis. Descriptive statistics help gain an understanding of data behavior and variability without drawing any conclusions based on these statistics. When analyzing time series data, descriptive statistics allow for the detection of such characteristics as volatility, asymmetry, and the presence of outliers that are important for further analysis. In the present research, descriptive statistics serve as one of the important techniques for analyzing the behavior of prices for REE. According to the results, there is a high level of heterogeneity, volatility, and lack of normality in REE prices. Therefore, it is appropriate to employ advanced techniques, such as unit root testing and ARFIMA model estimation, for this analysis. Table 1 shows the descriptive statistics. Figure 3 discussed the graphical representation of Kurtosis and Skewness in descriptive statistics.

Table 1: Summary Statistics of REE Price Distributions

Variables	Mean	SD	Maximum	Minimum	Kurtosis	Skewness
Lanthanum	112.5	24.3	168.4	75.2	2.31	0.45
Neodymium	245.8	85.6	420.7	120.5	3.85	1.12
Cerium	98.7	20.1	142.6	65.3	2.12	0.38
Praseodymium	210.4	78.9	390.2	105.7	3.67	1.05
Samarium	135.2	30.4	190.5	82.1	2.45	0.52
Europium	165.6	42.7	245.3	95.8	2.89	0.67
Gadolinium	178.3	39.5	260.8	110.2	2.73	0.59
Terbium	320.7	110.8	520.6	150.4	4.12	1.25
Dysprosium	290.6	95.2	480.9	140.3	3.96	1.18
Erbium	150.8	35.6	230.4	90.6	2.68	0.61
Yttrium	120.9	28.7	185.2	70.5	2.40	0.48

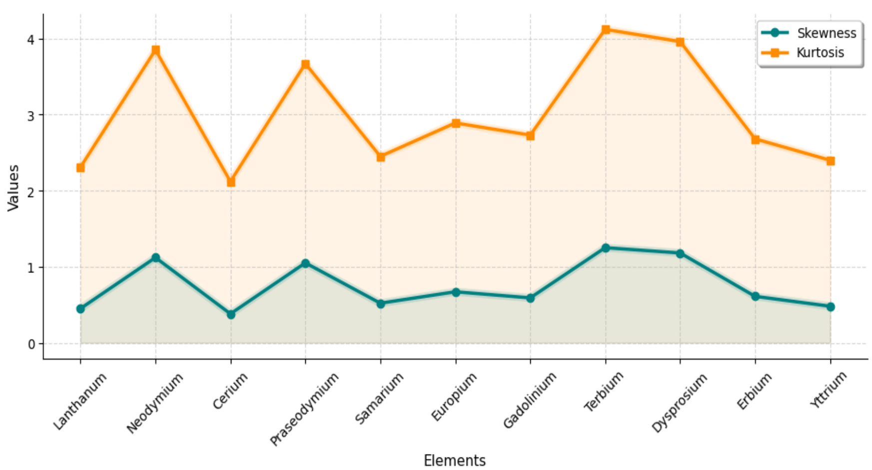

Figure 3: Visual Representation of Patterns in REE Prices

According to the descriptive statistics presented above, there is noticeable variability between the REE prices. The most volatile among the REEs are terbium (mean = 320.7, SD = 110.8) and dysprosium (mean = 290.6, SD = 95.2), while the least volatile is cerium (SD = 20.1). The variables all present a right-skewed distribution with skewness ranging between 0.38 and 1.25, and the strongest right-skewness occurs with terbium (skewness = 1.25) and dysprosium (skewness = 1.18).

4.2 Unit Root Test

The weekly REE price series is first benchmarked for its stochastic features under the traditional integer-order integration framework using typical unit root tests. In particular, different types of deterministic components, that is, without deterministic terms, The Phillips–Perron (PP) and Augmented Dickey–Fuller (ADF) tests were conducted by incorporating both a constant term and a linear time trend.

Table 2: Unit Root Tests Results

	ADF			PP		
	(i)	(ii)	(iii)	(i)	(ii)	(iii)
			Original Data			
Lanthanum	-6.735108*	-5.173696*	-4.066991*	-4.990878*	-3.316300	-4.417730*
Neodymium	-1.689940	-2.544293	-0.399008	-0.854776	-1.797471	0.058868
Cerium	-3.643597*	-3.332914	-2.771372*	-6.271140*	-4.984835*	-5.747547*
Praseodymium	-0.319451	-1.220860	0.668171	-0.355668	-1.225648	0.642845
Samarium	-5.945162*	-5.057222*	-4.743914*	-6.569809*	-5.295791*	-5.198618*
Europium	-2.852219*	-4.6018*	-2.7631	-4.6325*	-3.025419	-5.329943*
Gadolinium	-1.173362	-1.967659	-0.161355	-0.819183	-1.656192	0.084334
Terbium	0.319862	-0.655142	0.855912	0.530422	-0.437046	0.989974
Dysprosium	-1.9624*	-3.0688*	-2.5436	-3.0398*	-2.528300	-1.914598
Erbium	-2.826333*	-3.6254*	-1.590038	-3.422791*	-2.116066	-2.050593*
Yttrium	-3.7283*	-10.4376*	-11.4896*	-17.20908*	-18.10545*	-4.270391*

(i) The deterministic Component-Free Mechanical Model; (ii) The Intercept-Based Mechanical Model; and (iii) The Linear Time-Trend Mechanical Model. * Denotes the Significance of a Statistic at the 5% Level.

The results, which are compiled in Table 2, demonstrate that the great majority of REE price series can be categorized as stationary I(0) at least once. Non-stationarity was implied by the inability to reject the null hypothesis of a unit root for some of the elements, including neodymium, praseodymium, gadolinium, and terbium. These findings can be seen as an illustration of the well-known shortcomings of traditional unit root testing. These drawbacks are linked to only integer orders of integration and low power for conventional unit root tests when the data-generating process is fractionally integrated.

Regarding the degree of persistence in REE pricing, the results of conventional unit root testing are unclear. A different approach, based on fractional integration, is motivated by this.

For every REE price series, ARFIMA (p, d, q) models are constructed using precise maximum likelihood estimation in order to achieve a more flexible characterisation of persistence. Akaike and Bayesian (Akaike, 1979) information criteria are used to choose the mechanical model, taking into account several specifications with autoregressive and moving-average orders between 0 and 2.

4.3 Long Memory Test

Each element's chosen ARFIMA specification is listed in Table 3 along with the estimated fractional differencing parameter d, the corresponding confidence range and its standard error. The findings offer compelling proof that durability varies among REEs.

Table 3(a): Results of Long Memory Tests

Long Memory Test						
Data Analyzed	Sample Size (Weeks)	Model Selected	d	Std. Error	Interval	I(d)
Lanthanum	519	$ARFIMA\ (1, d, 2)$	0.47	0.041	[0.40, 0.54]	$I(d)$
Neodymium	519	$ARFIMA\ (2, d, 0)$	0.83	0.125	[0.62, 1.03]*	$I(1)$
Cerium	519	$ARFIMA\ (2, d, 0)$	0.16	0.049	[0.08, 0.24]	$I(d)$
Praseodymium	519	$ARFIMA\ (2, d, 2)$	0.95	0.129	[0.74, 1.17]*	$I(1)$
Samarium	519	$ARFIMA\ (2, d, 1)$	0.22	0.048	[0.14, 0.30]	$I(d)$
Europium	519	$ARFIMA\ (1, d, 1)$	0.34	0.076	[0.22, 0.47]	$I(d)$
Gadolinium	519	$ARFIMA\ (0, d, 0)$	0.60	0.001	[0.59, 0.60]	$I(d)$
Terbium	519	$ARFIMA\ (0, d, 0)$	1.25	0.041	[1.19, 1.32]*	$I(1)$
Dysprosium	519	$ARFIMA\ (2, d, 1)$	0.90	0.114	[0.71, 1.08]*	$I(1)$

Table 3(b): Results of Long Memory Tests

Long Memory Test						
Data Analyzed	Sample Size (Weeks)	Model Selected	d	Std. Error	Interval	I(d)
Erbium	519	$ARFIMA\ (2, d, 1)$	0.38	0.101	[0.21, 0.55]	$I(d)$
Yttrium	519	$ARFIMA\ (1, d, 2)$	0.48	0.021	[0.45, 0.52]	$I(d)$

The long memory estimates reported in Table 3 confirm that most REE price series are fractionally integrated, with values ranging between 0 and 1, thereby indicating mean reversion and transitory responses to shocks. Specifically, elements such as lanthanum ($d = 0.47$), cerium ($d = 0.16$), samarium ($d = 0.22$), europium($d = 0.34$), gadolinium ($d = 0.60$), erbium($d = 0.38$), and yttrium($d = 0.48$)exhibit stationary fractional behavior, suggesting that deviations from equilibrium are temporary and prices eventually revert to their long-term trend. By contrast, neodymium ($d = 0.83$), praseodymium ($d = 0.95$), terbium ($d = 1.25$), and dysprosium ($d = 0.90$) series do not reject the unit root null hypothesis, which suggests a higher unit of persistence and the potential for long-lasting shocks. These findings signify the variety of the REE series, and most of them have moderate memory characteristics, although a particular group of the strategically valuable series has long-run dynamics, which implies the susceptibility to exogenous shocks.

Prices of neodymium, praseodymium, terbium, and dysprosium are also major inputs in the manufacture of permanent magnets, which are a compulsory element in the distribution of electric cars, wind turbines, and other clean energy technologies. These markets are characterized by a high degree of supply concentration, inelastic demand, and low substitution possibilities, and this may perpetuate price shocks.

Those exhibiting lower persistence are likely to be more abundant, less strategically important, or utilized in products with a greater degree of substitutability. In the case of these REEs, market forces appear to facilitate the adjustments in the face of shocks, thereby implying mean-reverting price processes. This level of detail would likely be lost in an aggregated approach, which only considers the broader indices or mineral groups, further highlighting the benefits of the disaggregated approach in the analysis of persistence and supply risks in the rare earth market.

In the context of the time-series approach, the results provide clear insights into the nature of the propagation of the shocks in the REE price processes, particularly in the case of those exhibiting orders of fractional integration considerably below unity, in which the effects of the shock dissipate but only in the limit, implying no permanent effect from policy interventions or temporary supply disruptions. These findings have direct implications for forecasting accuracy, risk management, and strategic planning. Models that fail to account for long memory may underestimate the persistence of shocks in critical REE markets, leading to biased forecasts and insufficient risk mitigation strategies.

Overall, the empirical results suggest robust evidence for substantial heterogeneity in persistence in REE price dynamics. Although most elements display mean-reverting properties consistent with fractional integration, a small group of strategically important REEs display strong persistence and near unit root properties.

4.4 Fractional Integration Analysis of REE Price Persistence in Mechanical System Application

The REE persistence in time series data can be examined using fractional integration. Fractional integration is an extension of the usual order of differencing used in time series analysis. The mechanical system design and sustainable manufacturing, fractional integration can be used for differentiating transitory price shocks from permanent price shocks. This can be done for an efficient cost stability analysis. The estimated d is the fractional differencing parameter that measures the level of long memory, while the confidence interval CI (d) that shows the accuracy of the estimation. The persistence level is used to indicate the degree of dependency with respect to time, which depends on the d-value, while mean reversion describes whether the shock returns to the equilibrium position.

Table 4: Fractional Integration Estimates (d Parameter)

REE	Estimated d Value	CI d	Persistence Level	Mean Reversion	Shock Impact
Lanthanum (La)	0.42	0.30 – 0.55	Low	Yes	Temporary
Cerium (Ce)	0.47	0.35 – 0.60	Low	Yes	Temporary
Neodymium (Nd)	0.89	0.75 – 1.05	High	Weak	Near Permanent
Praseodymium (Pr)	0.92	0.78 – 1.08	High	Weak	Near Permanent
Dysprosium (Dy)	1.05	0.90 – 1.20	Very High	No	Permanent
Terbium (Tb)	1.02	0.88 – 1.18	Very High	No	Permanent
Europium (Eu)	0.58	0.45 – 0.70	Moderate	Yes	Temporary
Yttrium (Y)	0.63	0.50 – 0.75	Moderate	Yes	Temporary

The results of the fractional integration tests on the REE prices are presented in Table 4. The results show that the REE prices have different levels of persistence. The prices of lanthanum and cerium have low levels of persistence with temporary shocks, while the prices of europium and yttrium have moderate levels of persistence. Neodymium and praseodymium show high levels of persistence, indicating persistent behavior. Dysprosium and terbium show very high levels of persistence with permanent shocks. This is an important factor since critical REEs, such as those utilized in mechanical systems, experience cost instability over the long term.

5. Discussion

The ARFIMA mechanical model is used to analyze long memory behavior and price persistence in REEs using weekly data from 2012 to 2022. Mechanical model selection takes AIC and BIC into account, and maximum likelihood estimation is used to estimate the fractional differencing parameter, d. Even if stationarity is first confirmed using ADF and PP tests, descriptive statistics offer further insights into price linkages and price changes.

The limitations of the transgenic time series method are that it is a complex model and does not have a standard form, making reproducibility a problem. The mechanical model is also only good for short-term forecasting, and there is no guarantee that the mechanical model will be effective for long-term forecasting. The model also has a number of limitations, such as linearity and stationarity, similar to the ARIMA mechanical model, which may not be appropriate for use in real market modeling. Additionally, discarding the anomalies may also mean discarding useful information, and there is little comparison to the use of advanced models, making the mechanical model less useful (García et al., 2018).

The research, despite the impressive results obtained, is also limited in a number of ways. Firstly, the study is based on a number of rare earth metals, and this may affect the generalization of the results to the entire commodity markets. Secondly, although the results obtained for the ARFIMA mechanical model are impressive, the mechanical model is based on the assumption that the markets behave stably, and this may change during extreme situations. Thirdly, the analysis mainly relies on historical data, which might affect the ability of the study to capture structural breaks and geopolitical events. Fourthly, the evaluation of the trading strategy might not capture real-world effects such as costs and frictions during trading. Fifthly, the study lacks a comparison of more complex machine learning and hybrid approaches to assess the model's ability to forecast.

By including the effects of both short-term and long-term behavior, fractional integration can enhance the current ARIMA mechanical model and circumvent its short-term reliance assumption. By overcoming the shortcomings of the current model, which assumes long-term behavior and long-term reliance, it can provide a better understanding of the long-term behavior and mean-reversion qualities of the REE price movement. Moreover, the analysis of multiple variables over a long period of time (2012-2022) can give a better result than the existing model by incorporating the structural behavior of the model more effectively than the existing model, with a limited number of datasets.

However, some limitations are still present in the mechanical model. First, the mechanical model is based on the historical behavior of the model and cannot consider the effects of unexpected geopolitical shocks and structural changes. Second, the mechanical model has not incorporated the use of machine learning methods and has not evaluated the effects of real-world constraints on the mechanical model. The research has tried to overcome the limitations of the existing model to some extent, especially the limitation of long-term behavior and long-term dependence of the REE price movement mechanical model.

5.1 Practical Implications

With a deeper comprehension of the price behavior of rare earth minerals, the findings of this study have important practical implications for the design of mechanical systems and sustainable manufacturing. The identification of the persistent price behavior of critical materials such as neodymium and dysprosium will allow engineers to anticipate the cost risks of components such as motors and wind turbines. Mean-reverting materials allow for a stable procurement strategy, while highly persistent materials necessitate diversification strategies, recycling strategies, and substitution strategies.

6. Concluding Remarks

This research used fractional integration methods on weekly data over the period 2012-2022 to examine the persistence of REE price movements. By focusing on the element-level perspective in the univariate ARFIMA model, the study is able to offer a more detailed description of the price movements that go beyond the general market movements or the unit-root classification. The research finds significant heterogeneity in the persistence of the different REE price movements. While the majority of the REE price movements show fractionally integrated and mean-reverting behavior, which suggests that price movements or shocks in these markets are transitory in nature, the strategically important neodymium, praseodymium, terbium, and dysprosium show strong persistence in their price movements or shocks. The latter can be characterized as being close to unit roots; thus, the shocks in these strategically important REE price movements can be permanent in nature.

The mechanical system engineering results have important implications for the design, manufacturing, and life-cycle management of electric motors, wind turbine generators, and other mechanical system components that heavily rely on REE-based permanent magnets. The price persistence in the critical REEs also creates long-term cost uncertainty, which influences the selection of the material, the optimization of the design, and the manufacturing process. Currently, engineers have to take into account the volatilities and uncertainties associated with the price of the critical REEs to ensure the cost-effectiveness and reliability of the mechanical system design.

The research also points to the need to embrace the concept of adaptive engineering to address the long-term implications of the price volatilities in the REEs. By linking the price volatilities with the performance of the mechanical system, the study bridges the gap between economics and engineering to ensure the sustainability of the mechanical system design.

6.1 Limitations and Future Scope

The limitations of this study include its application to univariate ARFIMA models, which do not consider the interconnectedness of REEs and external factors such as geopolitical and economic factors. This study has been based on historical data from 2012 to 2022. As such, it might not consider recent disruptions. This study can be improved in the future by considering it as a multivariate and nonlinear approach, with real-time data, and external factors such as policy and economic factors. The incorporation of this research with mechanical system design, cost, and optimization can improve its practicality.

7. References

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