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Abstract

This paper uses fractional integration methods to measure the degree of persistence in historical annual data on the world population over the period 1800-2016. The analysis is carried out for the original series, and also for its log transformation and its growth rate. The results indicate that the series considered are highly persistent; in particular, the estimated values of the fractional diffencing parameter are above 1, which implies that shocks have permanent effects. Endogenous break tests detect one main break shortly after WWII. The evidence based on the corresponding sub-sample estimation indicates a sharp fall in the degree of dependence between the observations in the second sub-sample. Although the original data and their log transformation still exhibit explosive behaviour in that sub-sample, the growth rates are mean-reverting, and thus shocks to these series will only have transitory effects; moreover, there is a negative time trend. This has implications for the design of policies aimed at containing population growth.

JEL-Codes: C220, C400, J110.

Keywords: population growth, long memory, fractional integration, time trends.

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1. Introduction

The world population has increased sharply over the history of the planet. 12,000 years ago, it was only 4 million, which would now be the size of a city. Currently, it is 1860 times larger than at that time (see https://ourworldindata.org/world-population-growth). Its most significant growth has occurred in modern times: its size was still under 1 billion at the beginning of the 19th century (Kremer, 1993); it then increased sevenfold, the current population representing 6.5% of the total number of individuals born during the entire history of mankind, which was estimated to have been 108 billion (Haub, 1995). Growth was particularly rapid between 1950 and 1987, when the world population increased from 2.5 to 5 billion, the highest growth rate (2.1%) being recorded in 1962; since then, growth has decelerated, though it remains fast (Roser et al., 2013).

It should be noted that growth is driven by the difference between births and deaths. Most recently, the increase in deaths has not been matched by a similar one in births, which implies that the world population growth may halt in the near future. The 'demographic transition' model (Kirk, 1996) explains how growth occurs by identifying five different stages, namely: (i) Stage 1: mortality and birth rates are both high; (ii) Stage 2: mortality falls but birth rates are still high; (iii) Stage 3: mortality is low and birth rates fall; (iv) Stage 4: mortality and birth rates are both low; (v) Stage 5: mortality is low and there is some evidence of rising fertility (when the fertility rate is lower than two, the population decreases in the long run - Roser et al., 2013).

The present study provides evidence on the degree of persistence of the world population. This is measured using a fractional integration framework, where the fractional differencing parameter is the estimated persistence. This approach is more general than standard ones based on the I(0) stationary versus I(1) nonstationary dichotomy since it allows the order of integration to take any real values, including fractional ones. As a result, it encompasses a much wider range of stochastic processes and sheds light on whether or not the series is mean-reverting (and thus whether the effects of shocks are transitory or permanent) and the speed of the dynamic adjustment towards the long-run equilibrium. This method is applied below to analyse the stochastic properties of a world population series starting in 1800, thus obtaining an interesting set of results with important policy implications.

The layout of the following: Section 2 briefly reviews the literature on world population trends; Section 3 describes the data and the empirical results: Section 4 offers some concluding remarks.

2. Literature Review

There exist a number of studies aiming to explain the observed trends in the world population. For instance, Caswell (1978) analysed how small changes in the probabilities of birth, growth, survival, and migration affect population growth (Caswell, 1978). Specifically, he showed how, in a system of equations in linear differences, the biggest eigenvalue corresponds to the speed of population growth. A similar approach was used by Hamilton (1966), Emlen (1970) and Goodman (1971) for modelling the world population by age groups. By contrast, Tuljapurkar and Orzack (1980) considered instead a Markov process with a Leslie matrix for each time interval, and concluded that the world population is log-normal, which is consistent with models of geometric growth including non-negative growth. A logistic model was instead estimated by Marchetti et al. (1996) to capture the behaviour of both life expectancy and fertility; however, they could not reach definite conclusions regarding the future path of the world population.

Stochastic demography models have provided new insights into the likely effects of increased environmental variability on population trends (Boyce et al., 2006). Lutz and Quiang (2002) focused instead on the factors that can affect population trends by causing a decrease in procreation in the long run. Birdsall (1988) investigated the relationship between economic development and population growth and showed the importance of migration and urbanisation as drivers of demographic change. Gil-Alana et al. (2022) examined the same issue applying fractional integration and cointegration methods to historical data for Australia, Chile, Denmark, France, the UK, Italy, and the US from 1820 onwards. They found that the GDP and population series are highly persistent, but the evidence on the existence of a long-run equilibrium relationship linking these two variables is mixed, cointegration only holding in the cases of France, Italy and the UK. Finally, Climent and Meneu (2004) provided evidence of a linkage between the total fertility rate and GDP by estimatig vector error correction models and carrying out Granger causality tests.

With the exception of Gil-Alana et al. (2022), none of the above mentioned papers examines the degree of persistence of population data using a fractional integration approach. The present study applies the same method to historical data on the world population rather than on the population in individual countries as Gil-Alana et al. (2022) do and thus provides novel evidence.

3. Data and Empirical Analysis

The annual world population series used for the analysis spans the period from 1800 to 2016 and has been obtained from the 'OurWorldinData, which is a project of the Global Change Data Lab, a non-profit organisation based in the UK (Registered Charity Number 1186433), and is available from the following website: https://ourworldindata.org/world-population-growth#how-has-world-population-growth-changed-over-time.

FIGURE 1 ABOUT HERE

Figure 1 displays the evolution over time of the first differenced series. It can be seen that it increased gradually from 1800 till the beginning of the 20th century. It then experienced a sharp decline during both the First and the Second World Wars, after which it rose sharply, peaking in the 1980s, before subsiding as a result of a fall in fertility.

We analyse the behaviour of the world population by estimating a model with deterministic terms as standard in the unit root literature (Bhargava, 1986), namely:

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, \ldots, \tag{1}$$

where y_t stands for the series of interest, and β_0 and β_1 are the intercept and the (linear) time trend coefficient;[1] however, unlike in the standard unit root model, in our fractional integration framework the error term x_t is assumed to be integrated of order d, where d can take any real value, including fractional ones, i.e.,

$$(1 - B)^d x_t = u_t, \qquad t = 1, 2, \ldots, \tag{2}$$

Using a Binomial expansion, one can re-write equation (2), where B is the lag operator, for instance, $B^k x_t = x_{t-k}$, and u_t is I(0) (see Granger and Joyeux, 1980; Granger, 1980; 1981; Hosking, 1985), as follows:

$$(1 - B)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j B^j = 1 - dB + \frac{d(d-1)}{2} B^2 - \cdots, \tag{3}$$

where the higher the value of *d* is, the higher is the degree of association between observations distant in time. Note that if d = 0 the process exhibits short memory, whilst d>0 implies long memory; if d < 0.5, it is covariance stationary and mean reverting; if 0.5 ≤ d < 1 it is nonstationary but mean reversion still occurs; if d ≥ 1, the process is explosive.

We then implement the Lagrange Multiplier (LM) test using a version of the Whittle procedure in the frequency domain as in Robinson (1994) for the following null hypothesis:

[1] A quadratic term was found to be statistically insignificant.

$$H_o : d = d_o , \tag{4}$$

(for the empirical properties of this test, see Gil-Alana and Robinson, 1997; Gil-Alana and Moreno, 2012; Abbritti et al., 2016; etc.).

Three model specifications are considered, namely without deterministic terms, with an intercept only, and with an intercept as well as a linear time trend. Table 1 displays the estimates of d alongside their 95% confidence intervals, for both the original and the log-transformed data, under the assumption of white noise residuals, whilst Table 2 presents the results when allowing for autocorrelation in the error term u_t,; in both cases the coefficients in bold are those from the specification selected on the basis of the statistical significance of the regressors. Note that for the case of autocorrelated residuals we use the exponential spectral model of Bloomfield (1973), which is well suited to the framework proposed by Robinson (1994) and applied in this study. This specification approximates AR structures in a non-parametric way, and results in rapidly decaying autocorrelation coefficients (see, e.g., Gil-Alana, 2004).

TABLES 1 - 3 ABOUT HERE

Concerning the results with white noise residuals (Table 1), it can be seen that the time trend is not statistically significant, and the estimated value of d is greater than 1 for both the original data (1.46) and their log transformation (1.78). As for case of (Bloomfield) autocorrelation in the error term, the results are quite similar, though the estimates of are slightly lower (1.41 for the original data, and 1.71 for the logged ones). We also conducted the analysis for the growth rate, calculated as the first difference of the logged values (Table 3). The parameter d is now estimated to be equal to 0.78 with white noise errors and 0.65 with autocorrelated ones, with the unit root null hypothesis being rejected in the former case in favour of mean reversion (d < 1), but not in the latter one.

TABLES 4 AND 5 ABOUT HERE

Given the long time span, it is possible that breaks have occurred. Therefore we carry out the Bai and Perron’s (2003) break tests. These results are reported in Table 4. Three breaks are detected in the case of the original data (1915, 1948 and 1981) and five in the case of the logged ones (1832, 1880, 1915, 1948 and 1981). The same number of breaks (and break dates) is found in both cases for the growth rates, which are calculated as the first differences of the logged series. However, splitting the sample accordingly would yield very short subsamples with unreliable estimates. Therefore, we carry out the tests again allowing for a single break only. This appears to have occurred in 1948 in the case of the original data, and in 1946 for the logged series and the growth rate (Table 5).

TABLES 6 - 8 ABOUT HERE

Tables 6, 7 and 8 report the estimated values of d corresponding to the two subsamples based on the detected breaks for each of the three series (original data, log-transformed ones, growth rates), again for the three specifications without deterministic terms, with an intercept only, and an intercept as well as a linear time trend. It is noteworthy that in the case of the original series (Table 6) there is a substantial reduction in the degree of integration after the break, the estimated value of d decreasing from above 2 (or even 3) before the break to 1 or around 1 after it. Similar evidence is obtained when using the logged values (Table 7), namely the degree of integration falls sharply after the break; in addition, there is now a significant positive trend in the second subsample. Finally, in the case of the growth rates (Table 8) there is a decrease in the degree of integration from the first to the second subsample (from 2.66 to 0.52 with white noise errors and from 1.05 to 0.58 with autocorrelated ones), but the time trend is now negative and significant in the second subsample regardless of the specification for the error term.

4. Conclusions

This paper uses fractional integration methods to measure the degree of persistence in historical annual data on the world population over the period 1800-2016. The analysis is carried out for the original series, and also for its log transformation and its growth rate. The results indicate that the series considered are highly persistent; in particular, the estimated values of the fractional diffencing parameter are above 1, which implies that shocks have permanent effects.

It should be noted that these findings could be biased in the presence of structural breaks which have been overlooked. Therefore we also carry out endogenous break tests which suggest that the main break in the data occurred shortly after the Second World War. The evidence based on the corresponding sub-sample estimation indicates a sharp fall in the degree of dependence between the observations in the second sub-sample. However, in the case of the original data and their log transformation they are still above 1, which implies explosive behaviour and permanent effects of exogenous shocks; in addition, there is a statistically significant positive time trend. By contrast, the growth rate of the world population, though not covariance stationary, is mean-reverting, and thus shocks to this series will only have transitory effects; moreover, there is a negative time trend. This represents important information for policy makers concerned with demographic trends, since it suggests that there are already some factors at work (such as a fall in fertility) slowing down growth in the world population; this should be taken into account when designing policies aimed at containing population growth owing to the limited resources of the planet.

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Figure 1: Time series plot

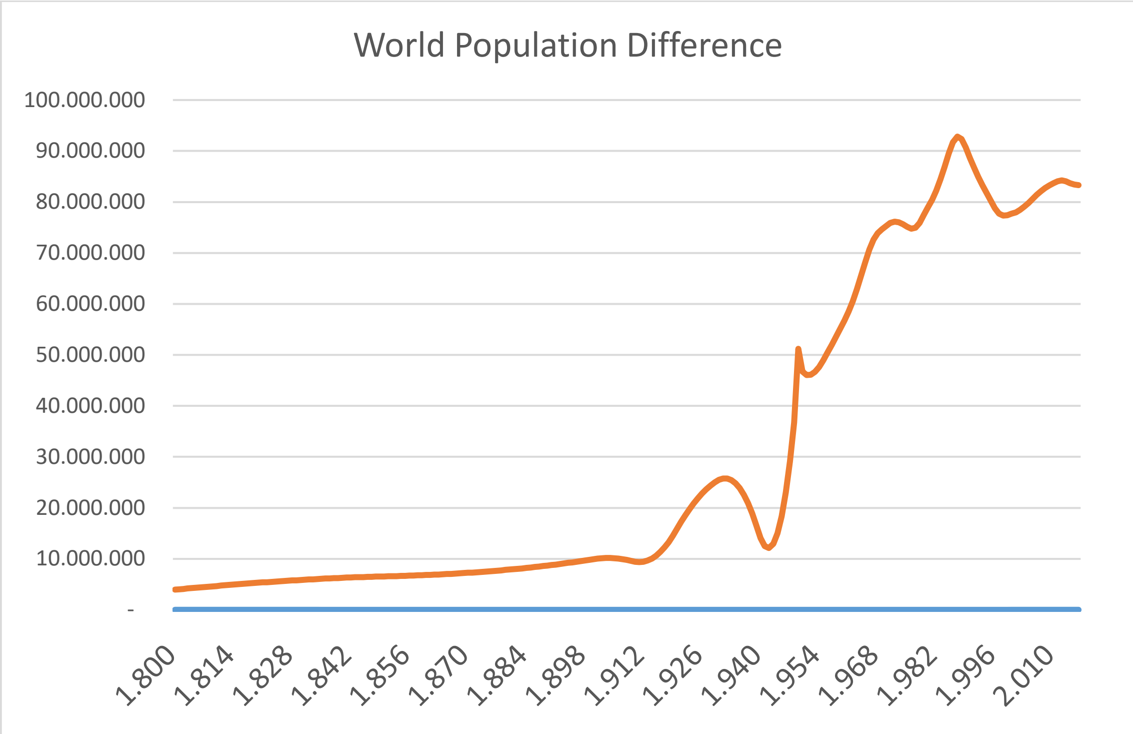

Table 1: Estimates of the differencing parameter, d - White noise errors

Series	No terms	With a constant	With a constant and a linear time trend
Original	1.44 (1.34, 1.57)	**1.46 (1.36, 1.59)**	1.46 (1.36, 1.59)
Log-transformed	0.98 (0.90, 1.10)	**1.78 (1.66, 1.92)**	1.78 (1.66, 1.92)

The values in bold are those from the model selected on the basis of the statistical significance of the regressors. The values in parenthesis are the confidence bands at the 95% level.

Table 2: Estimates of the differencing parameter, d - Autocorrelated errors

Series	No terms	With a constant	With a constant and a linear time trend
Original	1.38 (1.18, 1.72)	**1.41 (1.19, 1.75)**	1.41 (1.20, 1.75)
Log-transformed	0.95 (0.81, 1.15)	**1.71 (1.30, 2.20)**	1.71 (1.30, 2.20)

The values in bold are those from the model selected on the basis of the statistical significance of the regressors. The values in parenthesis are the confidence bands at the 95% level.

Table 3: Estimates of the differencing parameter, d, for the growth rate series

Series	No terms	With a constant	With a constant and a linear time trend
White noise	**0.78 (0.66, 0.92)**	0.78 (0.66, 0.92)	0.78 (0.66, 0.92)
Autocorrelation	**0.65 (0.30, 1.20)**	0.65 (0.30, 1.20)	0.65 (0.30, 1.20)

The values in bold are those from the model selected on the basis of the statistical significance of the regressors. The values in parenthesis are the confidence bands at the 95% level.

Table 4: Bai and Perron (2003) break test results

Series	N. of breaks	Break dates
Original	3	1915; 1948; 1981

Table 5: Bai and Perron (2003) break test results – one break only

Series	N. of breaks	Break dates
Original	1	1948
Log-transformed	1	1946
Growth rate	1	1946

Table 6a:Sub-sample estimates of the differencing parameter, d - Original data

i) White noise errors			
Series	No terms	**With a constant**	With a constant and a linear time
1800 - 1948	2.06 (1.95, 2.17)	**3.36 (3.21, 3.59)**	3.37 (3.21, 3.59)
1949 - 2016	1.18 (0.98, 1.49)	**1.14 (0.98, 1.38)**	1.13 (1.00, 1.35)
ii) Autocorrelated errors			
Series	No terms	With a constant	With a constant and a linear time
1800 - 1948	2.57 (1.85, 2.89)	**2.89 (2.62, 3.14)**	2.88 (2.62, 3.14)
1949 - 2016	0.65 (0.26, 1.02)	**1.20 (1.00, 1.45)**	1.16 (0.99, 1.41)

The values in bold are those from the model selected on the basis of the statistical significance of the regressors. The values in parenthesis are the confidence bands at the 95% level.

Table 6b: Sub-sample estimates of the coefficients from the selected models in Table 5a - Original data

i) White noise errors			
Series	No terms	With a constant	With a constant and a linear time trend
1800 - 1948	3.36 (3.21, 3.59)	3925.60 (14.43)	-----
1949 - 2016	1.14 (0.98, 1.38)	9737.60. (5.89)	----
ii) Autocorrelated errors			
Series	No terms	With a constant	With a constant and a linear time trend
1800 - 1948	2.89 (2.62, 3.14)	3925.62 (11.74)	-----
1949 - 2016	1.20 (1.00, 1.45)	9758.65. (5.38)	-----

The values in bold are those from the model selected on the basis of the statistical significance of the regressors. The values in parenthesis are the confidence bands at the 95% level.

Table 7a: Sub-sample estimates of the differencing parameter, d - Logged data

i) White noise errors			
Series	No terms	With a constant	With a constant and a linear time trend
1800 - 1948	0.99 (0.89, 1.14)	3.52 (3.07, 4.09)	**3.66 (3.15, 4.15)**
1949 - 2016	0.98 (0.83, 1.19)	1.46 (1.24, 1.76)	**1.39 (1.20, 1.65)**
ii) Autocorrelated errors			
Series	No terms	With a constant	With a constant and a linear time trend
1800 - 1948	0.93 (0.75, 1.21)	2.16 (1.09, 2.67)	**2.08 (1.07, 2.63)**
1949 - 2016	0.91 (0.62, 1.26)	1.09 (0.32, 1.59)	**1.08 (0.78, 1.63)**

The values appearing in bold indicate the significant model according to the deterministic components. The values in parenthesis are the confidence bands at the 95% level.

Table 7b: Sub-sample estimates of the coefficients from the selected models in Table 6a - Logged data

i) White noise errors			
Series	No terms	With a constant	With a constant and a linear time trend
1800 - 1948	3.52 (3.07, 4.09)	8.265 (184.63)	0.019 (2.27)
1949 - 2016	1.46 (1.24, 1.76)	9.456 (209.80)	0.055 (2.26)
ii) Autocorrelated errors			
Series	No terms	With a constant	With a constant and a linear time trend
1800 - 1948	2.08 (1.07, 2.63)	8.266 (138.84)	0.018 (2.05)
1949 - 2016	1.08 (0.78, 1.63)	9.998 (197.59)	0.020 (2.49)

The values in bold are those from the model selected on the basis of the statistical significance of the regressors. The values in parenthesis are the confidence bands at the 95% level.

Table 8a: Estimates of the differencing parameter, d - Growth rates

i) White noise errors			
Series	No terms	With a constant	With a constant and a linear time trend
1800 - 1948	2.29 (2.02, 2.81)	**2.66 (2.15, 3.14)**	2.66 (2.15, 3.15)
1949 - 2016	0.48 (0.29, 0.75)	0.41 (0.25, 0.68)	**0.52 (0.32, 0.73)**
ii) Autocorrelated errors			
Series	No terms	With a constant	With a constant and a linear time trend
1800 - 1948	1.40 (0.07, 1.82)	1.05 (0.06, 1.63)	**1.05 (0.05, 1.64)**
1949 - 2016	0.18 (-0.11, 0.70)	0.15 (-0.09, 1.00)	**0.58 (-0.06, 1.02)**

The values in bold are those from the model selected on the basis of the statistical significance of the regressors. The values in parenthesis are the confidence bands at the 95% level.

Table 8b: Estimated coefficients in the selected models in Table 7a - Growth rates

i) White noise errors			
Series	No terms	With a constant	With a constant and a linear time trend
1800 - 1948	2.66 (2.15, 3.14)	0.0187 (5.16)	-----
1949 - 2016	0.52 (0.32, 0.73)	0.1613 (4.36)	-0.0025 (-2.45)
ii) Autocorrelated errors			
Series	No terms	With a constant	With a constant and a linear time trend
1800 - 1948	1.05 (0.05, 1.64)	0.0173 (2.67)	0.0015 (2.21)
1949 - 2016	0.58 (-0.06, 1.02)	0.1761 (4.45)	-0.0027 (-2.16)

The values in bold are those from the model selected on the basis of the statistical significance of the regressors. The values in parenthesis are the confidence bands at the 95% level.