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Persistence and long run co-movements across stock market prices☆

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ABSTRACT

This paper investigates long memory, persistence and co-movements in the most representative stock markets from all over the world. We look at seven stock market indices from Europe, Asia and North America, first individually, by looking at the order of integration of the series from a fractional point of view and comparing different sampling periods (daily, weekly and monthly) for the time period 2009-2020. Then, co-movements across the series are examined by looking at the differences between them. The results indicate that all the individual series are highly persistent, with orders of integration close to 1 in most cases; evidence of a small degree of mean reversion is found in the two American indices (S&P500 and Dow Jones) and, generally, lower orders of integration are found at lower sampling frequencies. Focusing on the co-movements across the series, we observe a reduction in the degree of persistence in the one-by-one differential comparison of the series. Even though the differencing parameter is small compared with what we should have obtained under standard cointegration, this factor still shows long-memory as it ranges in the interval (0.5, 1) in the majority of cases; and appears to be greater when comparing markets from the same geographic region, showing evidence that the convergence process between the stocks is slower between markets of the same continents.

1. Introduction

The WWWF (World Wide Web Foundation) was established in 2009 by Sir Tim Berners-Lee and Rosemary Leith to advance the open web as a public good and a basic right. This open network, widely known today as the Internet, was first proposed in 1989, and heralded the start of the Digital era. One clear consequence of the widespread development of this network is globalization as information is now exchanged all over the world, increasing trade between the main economic regions across the globe. Evidence from time-series and cross-section regressions shows the significant effect of the Internet on trade in recent years. According to Freund and Weinhold (2004), this evidence is consistent with a model in which the Internet reduces market-specific fixed costs of trade. Financial markets are not too distant from this process as international financial markets are becoming increasingly interconnected, with equities displaying a high degree of co-movement across countries (Caporale et al., 2016).

The global financial crisis of 2007 and its far-reaching effects brought about greater coordination between all countries in an effort to resolve the problems which had arisen, and in doing so the G20 group was created to lead this process. Financial policies were coordinated in all economic regions creating new national and international institutions and a “new global framework” was proposed in order to implement policy recommendations contained in the G20. The global financial crisis provided a unique opportunity to go beyond economic data and attempt to capture cross border financial data and other information that could assist international and national institutions to measure and manage financial risk more effectively (Moshirian, 2011). Frank and Hesse (2009) argued that central bank interventions had a statistically significant impact on easing stress in unsecured interbank markets during the first phase of the subprime crisis that began in July 2007.

Traditionally, financial market volatility was said to have long memory and some mean reverting properties. Poterba and Summers

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(1988) suggested that shocks to stock market volatility do not persist for long periods, facilitating its predictability. Other studies focusing on the 2007 financial crisis have also questioned these properties, making a distinction between different periods in which these mean reversion properties do not hold. Thus, for example, Gil-Alana et al. (2014) investigated bull and bear periods, finding no evidence of systematic patterns in terms of persistence, though in some cases higher degrees of dependence were found during bull periods. In this line, Caporale et al. (2016) analyzed the linkages between stock markets with cointegration techniques to determine if there are diversification benefits from investing in different stock markets. Defining d as the degree of integration, processes with d = 0 or I(0) are associated with covariance stationary (short memory) processes, while if d > 0, time series are said to be 'long memory', because of the strong association between observations far apart in time. In particular, if d belongs to the interval (0, 0.5), time series are still covariance stationary, while d ≥ 0.5 implies nonstationarity. Finally, if d < 1, the series are mean-reverting, implying that the effect of the shocks disappears in the long run, in contrast to what happens if d ≥ 1, when the effects of shocks persist forever.

Globalization and interconnection between markets are leading to a different challenge, seeking market co-movements and determining in which direction are they moving. Several works have been developed in the last decade for specific regions, with strong evidence of co-movements across the markets. However, for some periods and regions it has been found that these co-movements are not strong or even not bi-directionally generated as has been shown in Gagnon et al. (2016), Caporale et al. (2016), Todea (2016) Yavas and Dedi (2016), Cardona et al. (2017), Aktan (2018) and others.

The objective of this paper is to evaluate the market persistence properties for the US, European and Asian regions by studying the performance of stock market prices across the indices, checking if the effect of different sample collections (monthly, weekly and daily) matters within the selected time-frame. Thus, we will first study the degree of persistence of the individual markets, as this is of interest if we want to know if shocks in the series have transitory or permanent effects. In addition, we investigate if the degree of persistence in the series changes with the data frequency and fractional cointegration in order to investigate long run equilibrium relationships between the variables of interest. As in most previous studies there is little evidence of mean reversion in the individual indices (DePenya & Gil-Alana, 2004; Tabak, 2007; Narayan, 2008; Hasanov, 2009; Gozbasi et al., 2014; Tiwari & Kyophilavon, 2014; etc.). We will try to verify this property by studying mean reversion in the one-to-one differential comparisons, evaluating the possible co-movements between all the indices. Therefore, the contributions of this work are as follows: we first investigate persistence individually in the stock market series of seven European, Asian and American markets by using fractional integration. The study is conducted at different data frequencies to see if persistence is affected by it. Finally, long run equilibrium relationships between the indices are examined by using fractionally cointegrated methods in a vis-à-vis relationship among them. The results in this work indicate little evidence of mean reversion in the price indices individually, along with a reduction in the order of integration at lower frequencies. On the other hand, we find some degree of mean reversion in some cases in the differences between the indices, supporting the idea of long run co-movements between them.

The rest of the paper is structured as follows: Section 2 includes a short literature review on modeling stock market prices. Section 3 is devoted to the methodology. Section 4 describes the dataset, while Section 5 displays the main empirical results. Section 6 concludes the manuscript.

2. Literature review

Initial studies about the persistence of market fluctuations and long memory in stock prices can be found in the seminal paper of Mandelbrot and van Ness (1968), introducing the concept of Fractional Brownian Motion and Fractional Noise, and showing evidence of a long-term dependence structure of 200 daily samples in the SP500 index. Greene and Fielitz (1977) tested for long-term dependence in US stock returns, analyzing stock indices and firms' returns series to evaluate aggregation effects and demonstrating that many series are characterized by long-term dependence. Epps (1979) looked for correlations between price changes in common stocks of companies. They were found to decrease with the length of the measurement interval. According to this author this was due to the nonstationarity of security price changes, and also to the correlations between price changes in the same stock, and in different stocks in successive periods. Booth et al. (1982) studied persistent dependence in gold prices, observing a high degree of dependence in the series. Helms et al. (1984) studied theSP500 futures and defined the label of "memory" as the presence of dependency. In terms of stock market volatility, Poterba and Summers (1986) found that shocks to volatility tend to decay rapidly and therefore volatility shocks can have only a small impact on stock market prices. After this result, Fama and French (1988) and Poterba and Summers (1988) focused on the US market predictability, finding mean reversion properties. Pindyck and Rotemberg (1993) studied the co-movements of stock prices in companies with unrelated lines of business, finding evidence that these prices should move together only in response to changes in current or expected future macroeconomic conditions. Andersen and Bollerslev (1998) looked for more accurate volatility forecasts, showing that volatility models produce strikingly accurate interdaily forecasts for the latent volatility factor. Andersen et al. (2001) noticed that realized volatilities and correlations show strong temporal dependence and appear to be well described by long-memory processes, with strong evidence that realized volatilities and correlations move together in a manner broadly consistent with latent factor structure. A very extensive literature review of the different techniques followed for testing persistence structure and mean reversion can be found in Caporale et al. (2016). Some more recent approaches can be found in Jach (2017), that quantified time-varying comovements between international stock market returns for various countries, by making comparisons with cross-correlations with rolling-windows or multi-period returns; and in Zehri (2021) that followed a GARCH-Copula CoVaR approach to address the risk spillovers from the US to China, Japan, Hong Kong, and South Korea stock returns, with evidence of large spillover effects from the US to East Asian stock markets, becoming stronger during the COVID-19 period.

Regarding recent research on persistence in stock markets, Los and Yu (2008) studied the Chinese market using Hurst exponents, finding evidence of persistence in all the analyzed series. Cunado et al. (2009) analyzed with fractional integration the US stock market in the TI bubble period (1994-2002), finding a different degree of volatility persistence for bull and bear markets, and suggesting that the higher uncertainty during bear markets could be related to the higher persistence observed in bear markets. Chandra Pati and Rajib (2010) using GARCH models for Indian future markets between 2004-2008, also found evidence of clustering and high persistence. McMillan and Thupayagale (2011) examined African stock markets using GARCH models and found that volatility persistence was overestimated if structural breaks were taken into account. Hung-Chun et al. (2012) used daily samples between 2002 and 2008 for SP500 depositary receipts, demonstrating that different GARCH-type models can be used to forecast both volatility and VaR (value-at-risk). Bentes (2014) analyzed the G7 group stock market indices with daily observations between 1999 and 2009 using FIGARCH models, finding evidence of long memory in the conditional variance. Yaya and Gil-Alana (2014)studied the Nigerian market with GARCH models, also noticing a different level of persistence between bull and bear market phases. Chuang (2015), for the Taiwanese market, used Granger-causality networks finding also long memory in the realized volatility. Other recent papers finding evidence of long memory in the volatility of stock markets include among others Assaf (2016), Jin (2017) and Gkillas et al. (2018). Caporale et al. (2020a, 2020b) used fractional integration to analyze persistence and non-linearities with structural breaks in various European stock markets, finding support for some degree of persistence and mean reversion across countries and subsamples without any clearly identifiable patterns. Adekoya (2021) also investigated the issue of long memory in 26 OECD countries using fractional integration and obtained evidence of mean reversion and long memory in 18 out of the 26 countries investigated. In another recent paper, Caporale et al. (2020a, 2020b) examined the Russian stock market and found no-persistence, mean-reversion and no-permanent effects of shocks.

On the other hand, the concept of co-movements in international stock markets was introduced by Beltratti and Morana (2006) when defining common long memory factor models with fractional cointegration. They studied the relationship between macroeconomic factors and stock market volatility and found evidence of a twofold linkage between stock markets and macroeconomic volatility. Other estimations of co-movements that used quantile regressions were found previously in Baur and Schulze (2005), showing that daily stock index returns display "some" contagion (defined as the crisis-specific co-exceedance not explained by the covariates for different quantiles), and that this is predictable within and across regions. Furthermore, contagion depends on a regional (world) market return and its volatility and is stronger for extreme negative returns than for extreme positive returns. There is also important research on the relationship between financial markets and other indicators, such as oil prices (Nath Sahu et al., 2014; Guesmi et al., 2016; Hamdam & Hamdam, 2019; Mokni & Youssef, 2019; Hou et al., 2019; Salisu & Gupta, 2020; Sarwar et al., 2020; Ehouman, 2020; etc.) and macro fundamentals (Conrad & Loch, 2014; Otieno et al., 2019; etc.), where results of cointegration indicate the existence of a long-term equilibrium relationship.

Dealing specifically on the co-movements across the indices, after the seminal contribution of Beltratti and Morana (2006) there have been important contributions in recent years. Gil-Alana et al. (2014) analyzed the bull-bear cyclical pattern observed in European, US and Asian markets with GARCH specifications. They show that there is not a systematic pattern in terms of the persistence degree, noting however a higher degree of dependence during the bull periods. Cardona et al. (2017) tested volatility transmission between US stock markets and the six largest Latin American stock markets using MGARCH-BEKK. They used daily frequency from 1993 to 2013 and found strong evidence of volatility transmission from US to the Latin American markets but not so in the opposite direction. Caporale et al. (2016) analyzed for the time period 1986-2013 the SP500 and the EuroStoxx50 indices using fractional cointegration methods, suggesting that cointegration does not hold over the full sample; however, there was evidence of it over the subsample from 1996-2009. Gagnon et al. (2016) investigated cointegration and financial market integration between five US and European equity indices using daily time series between 2003-2013; they found an increase in persistence and in the speed of adjustment in the 2007-2009 global crisis period, while outside of that period, the cointegration relationship was more fragmented, especially for higher-order moments. Todea (2016) investigated the dynamics of volatility persistence/market integration in the case of 20 emerging stock markets during the period 1999-2013 with daily observations and employing rolling windows, concluding that the emerging markets were not fully integrated with the world market. Yavas and Dedi (2016) studied the linkages between ETF (equity exchange traded funds) and their volatility transmission using MARMA and GARCH methodologies with daily samples between 2010 and 2015, finding strong evidence of volatility spillovers in four out of the five countries under study. Aktan (2018) studied the long-term relationship between BRICS and US stock markets by employing co-integration models and Granger causality tests on daily samples between 2011-2016, finding evidence of unidirectional causality from the US market towards the Russian, South African and Indian stock markets, while there is only a bidirectional causal relation between US and Brazil. Lyócsa and Horváth (2018) measured co-movements between the U.S. and the G7 stock markets using return co-exceedances models with daily data from 1999 to 2014, showing evidence of transmission from the U.S. to the other developed markets during both non-crisis and crisis periods, especially for sizable shocks. Finally, Budd (2018) analyzed weekly index returns from 2000 to 2014 collected from the S&P500 Index and the four largest equity indices in the Asia-Pacific market, using a VECH-MGARCH modeling approach. They found that all exchanges are well-integrated as the volatility in one market leads the volatility of the other markets in the Asian-Pacific region.

3. Methodology

Fractionally integrated methods are used in the empirical section of this paper. We choose this method since we are interested in describing the degree of persistence in the indices and in the vis-à-vis differences in order to know if there are co-movements across the indices.

The standard approach to determine the degree of persistence in time series data is to look at unit root tests to determine if the series is stationary I(0) (with shocks having transitory effects) or nonstationary I(1) (with a unit root and the series having permanent effects of shocks). However, all classical unit root methods (e.g., Dickey and Fuller (1979); Phillips and Perron (1988); Kwiatkowski et al. (1992); Elliot et al. (1996); etc.) have very low power if the alternatives are of a fractional form (see, e.g., Diebold & Rudebusch, 1991; Hassler & Wolters, 1994; Lee & Schmidt, 1996; etc.). This is the motivation for using fractionally integrated methods in this paper.

The fractional integration or I(d) approach consists of taking d-differences in a given series to render it stationary I(0) and where d can be any real value, thus allowing for fractional degrees of differentiation. In other words, we say that x_t follows an integration of order d process, i.e., $x_t \sim I(d)$ if it can be represented as

$$(1 - L)^d x_t = u_t, \ t = 1, \quad 2 \tag{1}$$

where L is the lag operator, i.e, $Lx_t = x_{t-1}$, and u_t is a short memory or I(0) process.[1] If d = 0 in (1), $x_t = u_t$, and x_t is said to be short memory as opposed to the long memory case that takes place with d > 0. It is said to be long memory because of the large degree of association between observations which are far away in time, noting that the polynomial in L in (1) can be expressed as:

$$(1 - L)^d = 1 - dL + \frac{d(d-1)}{2}L^2 - \dots .$$

Thus, the differencing parameter d must be taken as a measure of the degree of persistence in the data, the higher its value is, the higher the degree of the dependence in the data is. Moreover, it permits us to distinguish between mean reversion and lack of it in a more flexible way than the standard methods that only use the values 0 (for stationary series and mean reversion) and 1 (for non-stationarity and lack of it). In the context of real values of d, mean reversion occurs as long as d is smaller than 1, and the lower the

[1] A short memory or I(0) process is defined as a process where the infinite sum of autocovariances is finite, including thus the white noise case, but also stationary and invertible ARMA processes. On the contrary, long memory is defined as a process where the infinite sum of autocovariances is infinite, and the I(d) processes with d > 0 belong within this category.

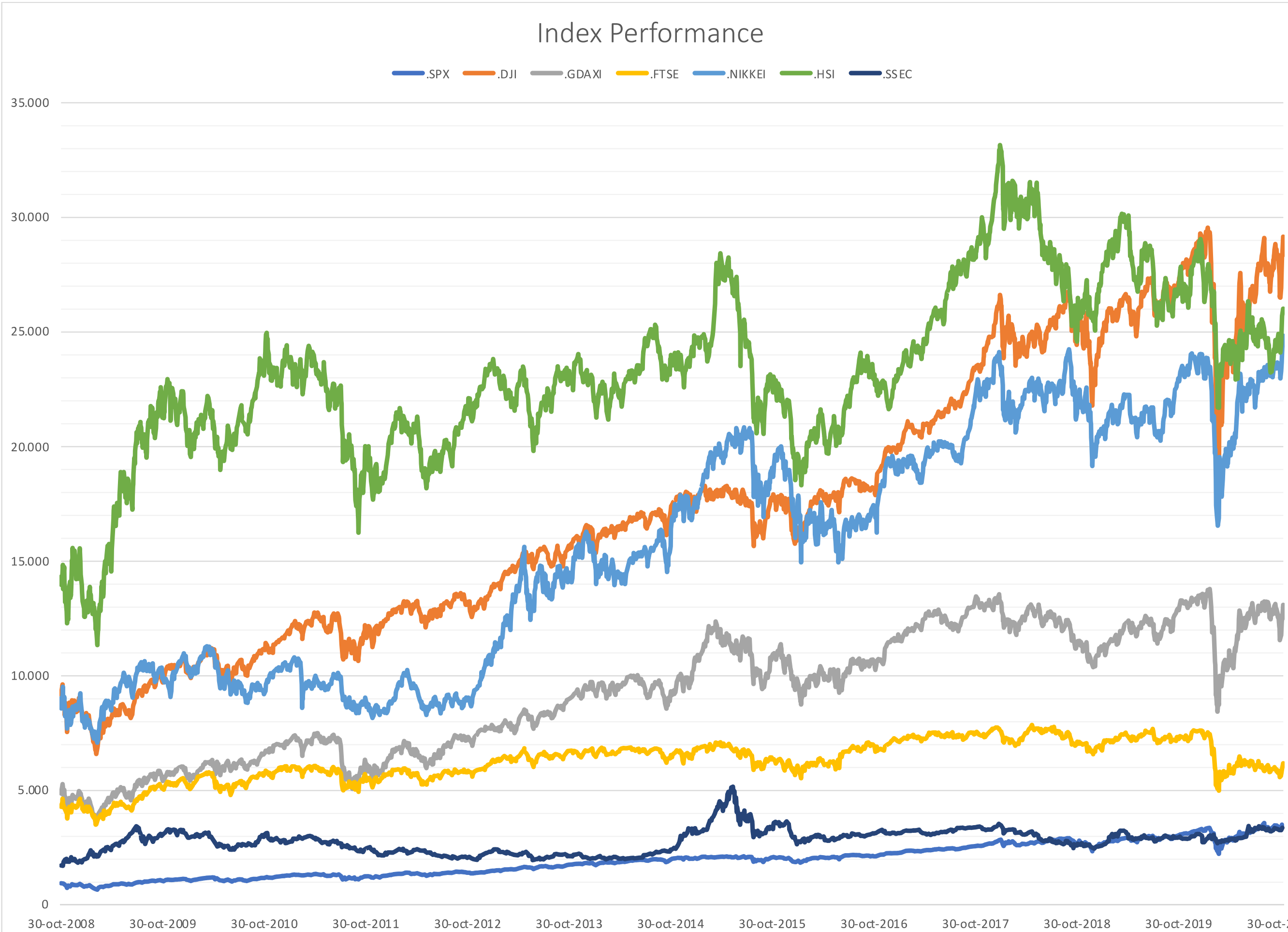

Fig. 1. Data series collected.

value of d is, the faster the process of convergence is to its original long-term projection.

Note that the differencing parameter d plays a crucial role in determining the nature of the time series under examination. Thus, if d = 0, the series is short memory, with shocks disappearing relatively fast (exponentially if the series displays autocorrelation of the AR type); if $0 < d < 0.5$, the series is long memory though still covariance stationary, with shocks lasting longer than in the previous case. If $0.5 \leq d < 1$, the series is no longer covariance stationary though it is still mean reverting with shocks disappearing in the long run; finally, if $d \geq 1$, the series is not mean reverting and shocks persist forever.

The estimation of d is conducted via the Whittle function in the frequency domain using a parametric approach developed in Robinson (1994). This is a testing procedure that tests the null hypothesis H_o: $d = d_o$, for any real value d_o in (1). Thus, it admits values in the nonstationary range ($d_o \geq 0.5$), and the limit distribution is standard normal independently of the assumptions made on the I(0) error term u_t, and even allowing for deterministic terms like an intercept and/or a time trend. In addition, it is the most efficient method in the Pitman sense against local departures from the null. (See Gil-Alana & Robinson, 1997, for the functional form of the version of the tests of Robinson, 1994 used in this application).

4. Data

We use data on a daily, weekly and monthly basis from seven major indices of US, European and Asian markets, over the period January 1st 2009 – November 9th 2020, avoiding thus the shock created by the 2007-08 financial crisis but covering the post-crisis and including the first wave of the COVID crisis. Specifically, we considered the S&P500 index (ISIN US78378X1072), the Dow Jones Industrial Average index (ISIN US2605661048), the Deutsche Boerse DAX index (ISIN DE0008469008), the Financial Times Stock Exchange 100 Index (ISIN GB0001383545), the Hang Seng Index (ISIN HK0000004322) and the Shanghai Stock Exchange Composite Index (ISIN CNM000000019) taken from the Reuters Eikon database, and the Nikkei 225 index (ISIN JP9010C00002) taken from the Bloomberg database. Figs. 1 and 2 summarize the collected data and their relative index performance, observing a certain co-movement across the indices.

Due to the existence of different regional labor calendars, a process of homogenization has been carried out for the daily sampled series for comparing the different data. In particular, as US markets were the largest ones in terms of capitalization, the US calendar has been followed for this analysis. For the other non-US indices, the non-US labor days have been removed and the previous closing value has been repeated in the case of foreign non-labor days. This is not an issue for monthly and weekly sampled series as there are samples for all ending weekly and monthly data over the period under analysis.

Table 1 displays the volatility coefficient for all indices across frequencies, while Fig. 3 shows the volatility coefficients along with CAGR returns. It can be observed that the observed volatility coefficient measured as the normalized standard deviation over the average return, grows as the sampling frequency grows. This

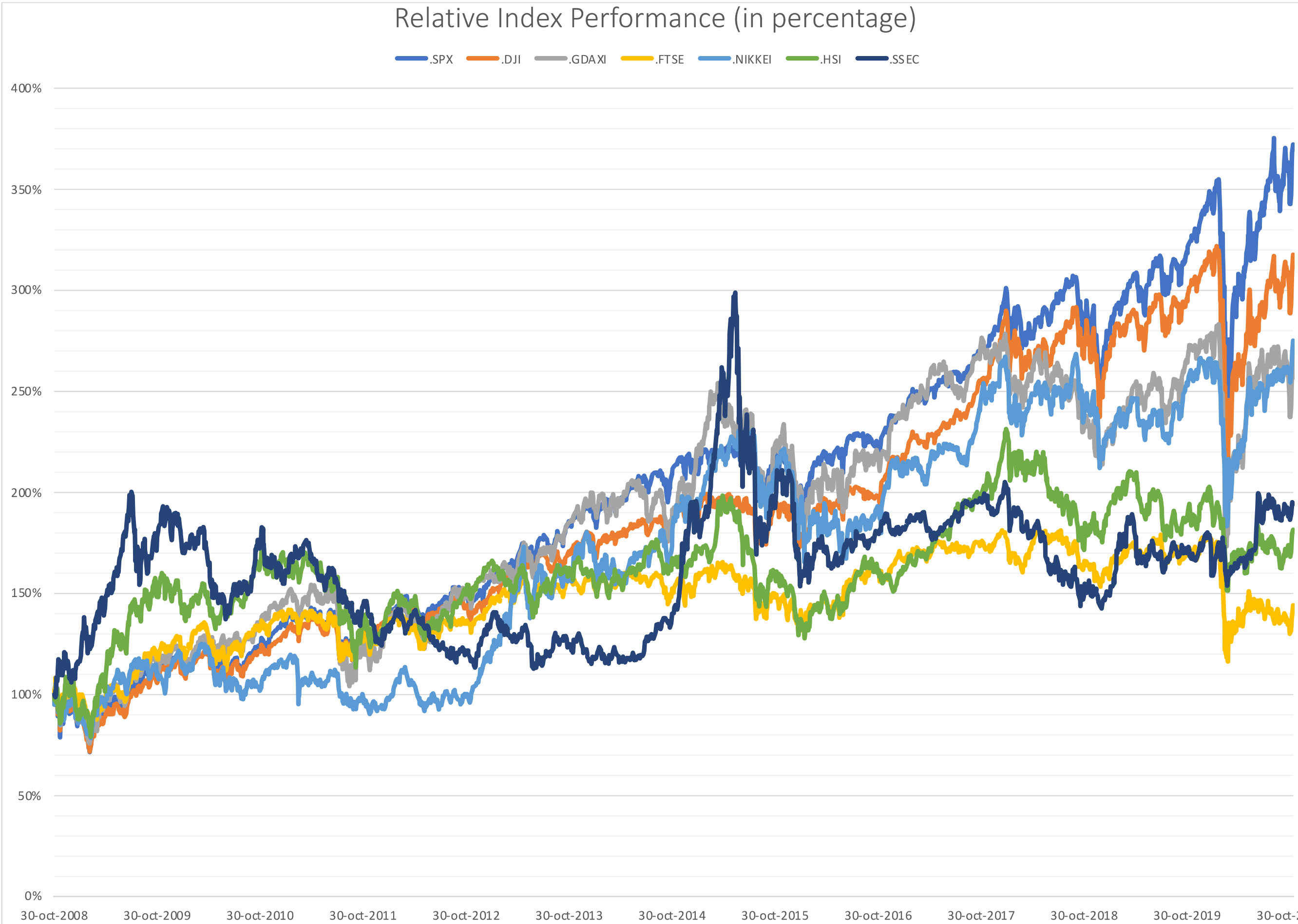

Fig. 2. Relative index performance.

volatility coefficient change ranges between 0.50% and 0.90% in all the series, therefore it may be more noticeable in the series with lower volatility such as the FTSE, with a daily normalized impact close to 5%. Regarding its relationship with the index return, as expected, greater volatility tends to give greater returns except in the case of the Shanghai market, which has greater volatility than the Hang Seng index but smaller returns, showing a worse Sharpe Ratio (Sharpe, 1994).

5. Empirical results

We start this section by estimating the fractional differencing parameter of each series separately. For this purpose, we consider the following model,

$$y_t = \beta_0 + \beta_1 t + x_t; \qquad (1 - L)^d x_t = u_t, \qquad t = 1, 2, ., \tag{2}$$

where y_t refers to the observed time series (price indices in logs) and we look separately at the cases of no deterministic terms (i.e., $\beta_0 = \beta_1$

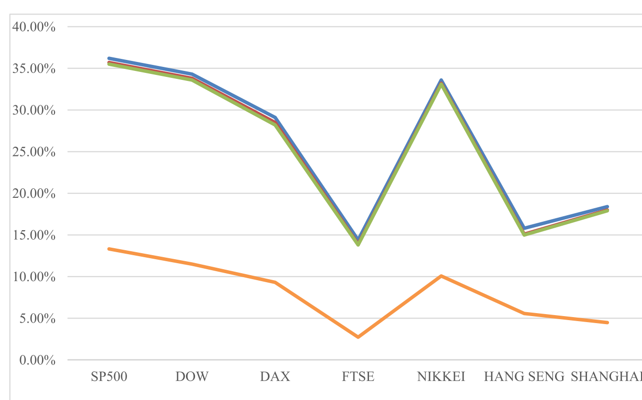

Fig. 3. Volatility coefficients and CAGR returns.

Table 1
Volatility coefficient (Standard Deviation/Average) on different sample periods.

Vol. Coefficient	SP500	DOW	DAX	FTSE	NIKKEI	H SENG	SHANGHAI	AVG
Daily	36,20%	34,30%	29,10%	14,40%	33,60%	15,80%	18,40%	25,97%
Weekly	35,70%	33,80%	28,50%	13,90%	33,20%	15,10%	18,00%	25,46%
Monthly	35,50%	33,60%	28,20%	13,80%	33,10%	15,00%	17,90%	25,30%
Daily-Monthly	0,70%	0,70%	0,90%	0,60%	0,50%	0,80%	0,50%	0,67%
Normalized Dailyv	1,93%	2,04%	3,09%	4,17%	1,49%	5,06%	2,72%	2,93%
Total return	295,94%	231,23%	166,38%	34,40%	187,43%	81,56%	61,98%	151,27%
CAGR	13,33%	11,50%	9,32%	2,72%	10,07%	5,57%	4,48%	8,14%

Table 2
Estimated coefficients in the logged transformed daily data.

DAILY	i) No autocorrelation		
Series	d	Intercept	Time trend
DAX	1.01 (0.98, 1.04)	8.5119 (635.19)	–
DOW JONES*	0.92 (0.90, 0.95)*	9.1054 (799.93)	0.0004 (3.68)
HANG SENG	1.00 (0.97, 1.03)	9.6180 (756.13)	–
NIKKEI	0.98 (0.96, 1.01)	9.0888 (662.77)	0.0003 (1.78)
FTSE	0.98 (0.95, 1.02)	8.4252 (768.15)	–
SHANGAI	1.02 (0.99, 1.05)	7.5387 (538.58)	–
S&P 500*	0.92 (0.89, 0.94)*	6.8341 (586.81)	0.0004 (3.93)
DAILY	ii) With autocorrelation		
DAX	0.98 (0.94, 1.04)	8.5109 (626.02)	0.0003 (1.71)
DOW JONES	1.05 (1.00, 1.11)	9.1100 (802.70)	–
HANG SENG	1.00 (0.94, 1.04)	9.6189 (756.82)	–
NIKKEI	1.00 (0.94, 1.04)	9.0892 (663.12)	–
FTSE	0.95 (0.91, 1.01)	8.4249 (769.96)	–
SHANGAI	1.01 (0.97, 1.05)	7.5390 (538.50)	–
S&P 500	1.03 (0.98, 1.07)	6.8373 (587.03)	0.0005 (1.83)

The values in parenthesis in column 2 are the associated 95% confidence bands for the values of d. In columns 3 and 4 they are t-values. * means evidence of mean reversion at the 5% level.

Table 3
Estimates coefficients on the logged transformed weekly data.

WEEKLY	i) No autocorrelation		
Series	d	Intercept	Time trend
DAX	0.97 (0.90, 1.04)	8.5095 (281.33)	–
DOW JONES*	0.90 (0.84, 0.98)*	9.1018 (380.77)	0.0019 (3.62)
HANG SENG	0.98 (0.93, 1.05)	9.6174 (354.49)	–
NIKKEI	0.97 (0.91, 1.04)	9.0860 (307.71)	0.0017 (1.71)
FTSE	0.94 (0.88, 1.02)	8.4225 (360.31)	–
SHANGAI	1.05 (1.00, 1.12)	7.5026 (252.55)	–
S&P 500*	0.91 (0.84, 0.98)*	6.8267 (286.51)	0.0021 (3.87)
WEEKLY	ii) With autocorrelation		
DAX	0.90 (0.79, 1.04)	8.5000 (284.17)	0.0016 (2.14)
DOW JONES*	0.85 (0.73, 0.99)*	9.0911 (385.33)	0.0019 (4.90)
HANG SENG	0.93 (0.84, 1.05)	9.6144 (356.09)	–
NIKKEI	0.91 (0.82, 1.01)	9.0860 (310.05)	0.0017 (2.45)
FTSE	0.89 (0.79, 1.00)	8.4189 (364.00)	–
SHANGAI	1.06 (0.97, 1.18)	7.5018 (252.72)	–
S&P 500*	0.83 (0.71, 0.98)*	6.8180 (292.81)	0.0021 (6.24)

The values in parenthesis in column 2 are the associated 95% confidence bands for the values of d. In columns 3 and 4 they are t-values. * means evidence of mean reversion at the 5% level.

Table 4
Estimated coefficients on the logged transformed monthly data.

MONTHLY	i) No autocorrelation		
Series	No terms	A constant	A linear trend
DAX	0.90 (0.77, 1.09)	8.3675 (160.13)	0.0071 (2.53)
DOW JONES*	0.80 (0.68, 0.97)*	8.9750 (231.30)	0.0087 (6.36)
HANG SENG	0.94 (0.82, 1.08)	9.5020 (176.69)	–
NIKKEI	0.97 (0.85, 1.13)	8.9793 (173.68)	0.0074 (1.97)
FTSE	0.93 (0.83, 1.07)	8.3320 (220.50)	–
SHANGAI	1.03 (0.90, 1.21)	7.5917 (114.24)	–
S&P 500*	0.79 (0.67, 0.96)*	6.7079 (171.70)	0.0097 (7.26)
MONTHLY	ii) With autocorrelation		
DAX*	0.64 (0.46, 0.86)*	8.4066 (189.81)	0.0075 (8.49)
DOW JONES*	0.61 (0.42, 0.85)*	8.9966 (2171.44)	0.0088 (14.53)
HANG SENG	0.96 (0.46, 1.31)	9.4988 (176.41)	–
NIKKEI	0.81 (0.63, 1.08)	8.9861 (180.55)	0.0075 (4.11)
FTSE	0.89 (0.66, 1.11)	8.3337 (221.94)	–
SHANGAI	0.75 (0.51, 1.04)	7.6653 (125.44)	–
S&P 500*	0.59 (0.34, 0.85)*	6.7405 (206.33)	0.0097 (17.12)

The values in parenthesis in column 2 are the associated 95% confidence bands for the values of d. In columns 3 and 4 they are t-values. * means evidence of mean reversion at the 5% level.

= 0 a priori in (2)); including only an intercept (i.e., β_1 = 0 a priori); and with an intercept and a linear time trend, choosing the more appropriate model by using the corresponding t- values of the estimated coefficients. This selection is based on the t-values of the coefficients in the d-differenced processes, noting that the two equations in (2) can be expressed as

$$\tilde{y}_t = \beta_0\, \tilde{1}_t + \beta_1 \tilde{t}_t + u_t, \qquad t = 0\,,1\,,\,., \tag{3}$$

with $\tilde{y}_t = (1-L)^d y_t$; $\tilde{1}_t = (1-L)^d 1$ and $\tilde{t}_t = (1-L)^d t$, and noting that u_t is I(0) by construction, the t-values on (3) remain valid.

As mentioned above, the estimation of the parameters in (2) is conducted via Robinson (1994), which is a very general testing procedure that allows us to consider model (2) as a particular case of his tests, displaying a standard normal limiting distribution, and holding for any real value d, and thus, not being restricted to the stationary region as is required with other procedures. Employing other parametric (Sowell, 1992; Beran, 1995) and semiparametric methods (Robinson, 1995; Phillips & Shimotsu, 2004; Shimotsu, 2010) produced essentially the same results.

Tables 2–4 report the estimated coefficients for d (and the associated 95% confidence bands), the intercept (β_0) and the time trend (β_1) respectively for the daily, weekly and monthly data. The upper part of the tables refers to the case of no autocorrelation, i.e., with white noise errors, while the lower panel reports the estimates under the assumption of autocorrelation. Here, we employ a non-parametric approach due to Bloomfield (1973). It is non-parametric in the sense that no explicit formula is displayed for the error term, being simply described in terms of its spectral density function, whose logged form approximated the one of AR structures.

We start with the daily data. Focusing first on the results based on no autocorrelation, we observe that all values of *d* are around 1.00, though we observe some differences across the series. Thus, evidence of mean reversion (i.e., estimates of *d* significantly below 1) are observed in the cases of the two American indices, Dow Jones and S&P500; however, for the remaining five cases, the null hypothesis of a unit root (i.e., *d* = 1 or I(1)) cannot be rejected. Thus, according to this simple model, the random walk hypothesis cannot be rejected in five out of the seven series examined, supporting in these cases a weak form of the Efficient Market Hypothesis (EMH). If autocorrelation is permitted (in the lower part of the table), the I(1) hypothesis cannot be rejected in any single case. Focusing on the estimated coefficients (Table 2b) we notice that a time trend is required for the Dow Jones, Nikkei and S&P500 with no autocorrelation, and for the Dax and S&P500 with autocorrelation. This significant (positive) trend, however, is not very relevant noting that the time trend becomes a constant if *d* = 1, and it tends to zero for *d* < 1.[2] As a conclusion, the results based on daily data indicate little support for the mean reversion hypothesis, finding evidence of this in only two of the US indices, and those to a very small degree (the coefficient for the differencing parameter is very close to 1, (0.92)) in both series

Next, we look at the weekly data (Table 3). Here, evidence of mean reversion is found again in the two American indices (Dow Jones and S&P500) and this occurs for the two cases of white noise and autocorrelated errors. Nevertheless, the values are again large and close to 1 implying a very long-lived effect of shocks and a very small degree of reversion to the mean. (The estimates of d are now 0.90 and 0.85 respectively for white noise and autocorrelation in the case of the Dow Jones, and 0.91 and 0.83 for the S&P500). Finally, using monthly data (Table 4) the values of *d* are smaller than in the previous tables, and mean reversion is now observed once more in the two American indices along with the Dax with autocorrelated errors. The estimates of d are now even smaller than in the previous cases, and d is equal to 0.80 for Dow Jones and 0.79 for S&P500 under the assumption of no autocorrelation for the error term, and 0.64 (Dow Jones), 0.61 (Dax) and 0.59 (S&p500) with autocorrelation.

[2] Note that if d = 1 in (2) or (3) with u_t as a white noise process, the model becomes a random walk with a drift.

Table 5a
Comparison between the estimates of d on the logged transformed data with no autocorrelation.

	DAX	DOW JONES	HANG SENG	NIKKEI	FTSE	SHANGAI	S&P 500
No terms							
Daily	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	(0.97, 1.03)	(0.97, 1.03)	(0.97, 1.03)	(0.97, 1.03)	(0.97, 1.03)	(0.97, 1.03)	(0.97, 1.03)
Weekly	0.99	0.99	0.99	1.00	0.99	1.00	0.99
	(0.94, 1.05)	(0.94, 1.05)	(0.94, 1.05)	(0.94, 1.06)	(0.94, 1.06)	(0.95, 1.06)	(0.94, 1.05)
Monthly	0.96	0.96	0.98	0.97	0.96	0.99	0.97
	(0.86, 1.10)	(0.86, 1.10)	(0.88, 1.12)	(0.87, 1.11)	(0.86, 1.10)	(0.89, 1.13)	(0.87, 1.10)
A constant							
Daily	**1.01**	0.92	**1.00**	0.98	**0.98**	**1.02**	0.92
	(0.98, 1.04)	(0.90, 0.95)	**(0.97, 1.03)**	(0.96, 1.00)	**(0.95, 1.02)**	**(0.99, 1.05)**	(0.89, 0.94)
Weekly	**0.97**	0.90	**0.98**	0.97	**0.94**	**1.05**	0.91
	(0.90, 1.04)	(0.84, 0.98)	**(0.93, 1.05)**	(0.91, 1.04)	**(0.88, 1.02)**	**(1.00, 1.12)**	(0.84, 0.98)
Mothly	0.89	0.78	**0.94**	0.97	**0.93**	**1.03**	0.75
	(0.75, 1.10)	(0.69, 0.97)	**(0.82, 1.08)**	(0.85, 1.14)	**(0.83, 1.07)**	**(0.90, 1.21)**	(0.66, 0.95)
A linear trend							
Daily	1.01	**0.92**	1.00	**0.98**	0.98	1.02	**0.92**
	(0.98, 1.04)	**(0.90, 0.95)**	(0.97, 1.03)	**(0.96, 1.01)**	(0.95, 1.02)	(0.99, 1.05)	**(0.89, 0.94)**
Weekly	0.97	**0.90**	0.98	**0.97**	0.95	1.05	**0.91**
	(0.90, 1.04)	**(0.84, 0.98)**	(0.93, 1.05)	**(0.91, 1.04)**	(0.88, 1.02)	(1.00, 1.12)	**(0.84, 0.98)**
Monthly	**0.90**	**0.80**	0.94	**0.97**	0.93	1.03	**0.79**
	(0.77, 1.09)	**(0.68, 0.97)**	(0.83, 1.08)	**(0.85, 1.13)**	(0.83, 1.07)	(0.90, 1.21)	**(0.67, 0.96)**

Table 5b
Comparison between the estimates of d on the logged transformed data with autocorrelation errors.

	DAX	DOW JONES	HANG SENG	NIKKEI	FTSE	SHANGAI	S&P 500
No terms							
Daily	0.99	1.00	1.00	1.00	0.99	1.01	0.99
	(0.95, 1.03)	(0.96, 1.04)	(0.95, 1.03)	(0.94, 1.03)	(0.95, 1.03)	(0.96, 1.05)	(0.95, 1.04)
Weekly	0.97	0.97	0.98	0.98	0.98	1.01	0.99
	(0.89, 1.08)	(0.90, 1.09)	(0.90, 1.08)	(0.90, 1.08)	(0.90, 1.08)	(0.91, 1.10)	(0.89, 1.09)
Monthly	0.97	0.95	0.95	0.96	0.95	0.96	0.96
	(0.79, 1.20)	(0.77, 1.19)	(0.79, 1.19)	(0.77, 1.21)	(0.77, 1.19)	(0.79, 1.20)	(0.79, 1.20)
A constant							
Daily	0.98	**1.05**	**1.00**	**1.00**	**0.95**	**1.01**	1.03 (0.98, 1.07)
	(0.93, 1.04)	**(1.00, 1.11)**	**(0.94, 1.04)**	**(0.94, 1.04)**	**(0.91, 1.01)**	**(0.97, 1.05)**	
Weekly	0.90	0.85	**0.93**	0.91	**0.89**	**1.06**	0.83 (0.72, 0.98)
	(0.78, 1.04)	(0.75, 0.99)	**(0.84, 1.05)**	(0.82, 1.01)	**(0.79, 1.00)**	**(0.97, 1.18)**	
Mothly	0.56	0.61	**0.96**	0.80	**0.89**	**0.75**	0.59 (0.52, 0.69)
	(0.48, 0.76)	(0.54, 0.74)	**(0.46, 1.31)**	(0.64, 1.09)	**(0.66, 1.11)**	**(0.51, 1.04)**	
A linear trend							
Daily	**0.98**	1.05	0.99	0.99	0.95	1.01	**1.03**
	(0.94, 1.04)	(1.00, 1.11)	(0.94, 1.03)	(0.94, 1.03)	(0.91, 1.01)	(0.97, 1.05)	**(0.98, 1.07)**
Weekly	**0.90**	**0.85**	0.93	**0.91**	0.89	1.06	**0.83**
	(0.79, 1.04)	**(0.73, 0.99)**	(0.85, 1.05)	**(0.82, 1.01)**	(0.80, 1.00)	(0.97, 1.18)	**(0.71, 0.98)**
Monthly	**0.64**	**0.61**	0.96	**0.81**	0.90	0.76	**0.59**
	(0.46, 0.86)	**(0.42, 0.85)**	(0.70, 1.28)	**(0.63, 1.08)**	(0.73, 1.10)	(0.53, 1.04)	**(0.34, 0.85)**

In general, a feature observed across these tables is that the estimated values of d decreases as we move from daily to weekly or monthly data, showing a direct relationship between the data frequency and persistence. Tables 5a and 5b show the comparison between the estimates of the differencing parameter d, with no autocorrelation and autocorrelation error adjustments between different sampling periods. It can also be seen that "Linear trend" calculations offer very similar results to the "constant" adjustments, however, there is no evidence of a strong relationship between d, its changes between data frequency and the volatility coefficient of the index.

This is consistent with the work by Caporale et al. (2013), among many others, which show that lower degrees of integration are associated with lower frequencies in the case of the US dollar / British pound spot exchange rate. We also support this evidence with the lower degrees of integration corresponding to the monthly data. In addition, a small degree of mean reversion is found in the two American indices (S&P500 and Dow Jones).

In what follows we look at the potential existence of long run co-movements between the series, by testing the order of integration in a one-by-one relationship between the variables. Here we focus only on the daily data as it provides the greater volatility coefficient, and on the d parameter. Table 6 refers to the case of uncorrelated (white noise) errors, while Table 7 allows for autocorrelation throughout the model of Bloomfield (1973).

Starting with the results based on uncorrelated errors, we observe that the time trend is statistically significant in nine cases. Apparently, this could be an indication of lack of convergence between the series; note, however, that this is not the case since the time trend coefficients tend to a constant as the order of integration approaches 1. Focusing on these orders of integration, mean reversion is now found in practically all cases, though the values of d are relatively high in all cases. The only two cases where the I(1) hypothesis cannot be rejected correspond to Dax - FTSE and Dow Jones – S&P500, that is in the European and US relationships, In all the other cases, the values are significantly below 1, the lowest values corresponding to Nikkei – S&P500 ($d = 0.74$), Dow Jones – Nikkei (0.75), FTSE – S&P500 (0.76), and Dax – Nikkei, Dow Jones – FTSE and Hang Seng – S&P500, with $d = 0.77$.

Allowing autocorrelation in the error term, the estimated values of d are slightly higher and weak mean reversion is found in a lower

Table 6
White noise errors between differential series.

Differential series	d	Intercept	A time trend
DAX – FTSE	1.02 (0.99, 1.05)	-0.0864 (-12.16)	—
DAX – DOW JONES	0.83* (0.81, 0.86)	0.5956 (58.58)	—
DAX – HANG SENG	0.85* (0.83, 0.88)	1.1112 (79.79)	-0.00015 (-1.82)
DAX - NIKKEI	0.77* (0.75, 0.79)	0.5842 (40.20)	—
DAX - SHANGAI	0.94* (0.92, 0.97)	-0.9685 (-53.79)	—
DAX – S&P500	0.82* (0.79, 0.84)	-1.6745 (-164.80)	0.00012 (2.35)
DOW JONES – S&P500	0.98 (0.95, 1.01)	-2.2716 (-880.93)	—
DOW JONES – FTSE	0.77* (0.75, 0.79)	-0.6754 (-78.60)	-0.00028 (-9.48)
DOW JONES – HANG SENG	0.79* (0.77, 0.81)	0.5199 (38.21)	-0.00021 (-4.09)
DOW JONES – NIKKEI	0.75* (0.73, 0.77)	—	—
DOW JONES – SHANGHAI	0.93* (0.91, 0.95)	-1.5641 (-94.68)	—
FTSE – HANG SENG	0.84* (0.82, 0.87)	1.1957 (97.64)	—
FTSE – NIKKEI	0.81* (0.79, 0.83)	0.6656 (48.38)	0.00023 (3.67)
FTSE – SHANGHAI	0.94* (0.92, 0.97)	-0.8826 (-53.30)	—
FTSE – S&P500	0.76* (0.74, 0.78)	-1.5951 (-183.97)	0.00032 (11.75)
HANG SENG – NIKKEI	0.92* (0.89, 0.94)	-0.5290 (-39.46)	—
HANG SENG – SHANGHAI	0.84* (0.82, 0.86)	-2.0679 (-117.66)	—
HANG SENG – S&P500	0.77* (0.75, 0.79)	-2.7919 (-207.37)	0.00027 (5.78)
NIKKEI – SHANGHAI	0.91* (0.88, 0.94)	-1.5457 (-81.21)	—
NIKKEI – S&P500	0.74* (0.72, 0.76)	-2.2611 (-161.49)	0.00010 (2.55)
SHANGHAI – S&P500	0.92* (0.90, 0.95)	-0.7087 (-42.90)	0.00027 (1.67)

*: Evidence of mean reversion (d < 1) at the 5% level.

Table 7
Autocorrelated errors between differential series.

Differential series	d	Intercept	A time trend
DAX – FTSE	1.00 (0.94, 1.04)	-0.0861 (-12.11)	-0.00022 (-1.71)
DAX – DOW JONES	0.95* (0.91, 0.99)	0.5666 (57.38)	—
DAX – HANG SENG	0.95* (0.91, 0.99)	1.1078 (78.32)	—
DAX – NIKKEI	0.93* (0.89, 0.97)	0.5792 (38.03)	—
DAX – SHANGAI	0.98 (0.94, 1.03)	-0.9712 (-53.84)	—
DAX – S&P500	0.92* (0.86, 0.97)	-1.6749 (-161.08)	—
DOW JONES – S&P500	0.93* (0.88, 0.97)	2.2714 (-885.34)	0.000045 (1.63)
DOW JONES – FTSE	0.91* (0.87, 0.96)	-0.6811 (-75.81)	-0.00029 (-3.52)
DOW JONES – HANG SENG	0.96* (0.91, 0.99)	0.5109 (36.37)	—
DOW JONES – NIKKEI	0.95* (0.91, 0.99)	—	—
DOW JONES – SHANGHAI	1.03 (0.97, 1.06)	-1.5718 (-95.22)	—
FTSE – HANG SENG	0.94* (0.88, 0.97)	1.1939 (95.39)	—
FTSE – NIKKEI	0.96* (0.92, 0.99)	0.6644 (47.18)	—
FTSE – SHANGHAI	0.99 (0.95, 1.03)	-0.8655 (-53.39)	—
FTSE – S&P500	0.89* (0.86, 0.94)	-1.5907 (-175.27)	0.00034 (4.66)
HANG SENG – NIKKEI	1.00 (0.96, 1.05)	-0.5294 (-39.40)	—
HANG SENG – SHANGHAI	0.95* (0.88, 0.99)	-2.0771 (-116.15)	—
HANG SENG – S&P500	0.93* (0.86, 0.98)	-2.7838 (-198.61)	0.00026 (1.76)
NIKKEI – SHANGHAI	0.99 (0.94, 1.03)	-1.5495 (-81.04)	—
NIKKEI – S&P500	0.93* (0.89, 0.98)	-2.2541 (-152.38)	—
SHANGHAI – S&P500	1.02 (0.97, 1.05)	-0.7012 (-42.45)	—

*: Evidence of mean reversion (d < 1) at the 5% level.

number of cases (in 14 out of the 21 cases presented) the lowest values corresponding now to FTSE – S&P500 (0.89), Dow Jones – FTSE (0.91) and Dax – S&P500 (0.92). Table 8 shows a final comparison that groups the cases under the different regions under study. It can be said that allowing autocorrelation errors renders the relationships more homogeneous, as the average value of *d* calculated with white noise errors is around *d*=0.85 with a volatility coefficient of near 10%. Allowing for autocorrelation errors helps the reduction in the dispersion of results with a volatility coefficient of near 4% with a *d* coefficient smaller though very close to one in all cases (*d* = 0.96).

We can conclude by saying that there is some evidence of comovements in the vis-à-vis relationship between the indices, observing some degree of mean reversion in the majority of the cases. Thus, in the event of an exogenous shock affecting any of these relationships, its effect will tend to disappear in the long run though it may take some time. We should note here that the analysis has been conducted on the differences between the indices (and not on the residuals of a regression of one index against another as might be the case in a proper analysis of cointegration). The reason is that using our approach we still rely on observed values rather than estimated ones that would have required the computation of critical values to determine the existence of mean reversion. Other recent approaches based on fractional cointegration (such as the FCVAR model of Johansen & Nielsen, 2010, 2012) can be employed on these and other data.

Table 8
Regional comparison of differential series.

	White noise errors				Autocorrelated errors			
Series	d	Min	Max	Std. Dev/ Average	d	Min	Max	Std. Dev/ Average
SAME REGION								
DAX – FTSE	1.02	0.99	1.05		1.00	0.94	1.04	
DOW JONES – S&P500	0.98	0.95	1.01		0.93	0.88	0.97	
HANG SENG – NIKKEI	0.92	0.89	0.94		1.00	0.96	1.05	
NIKKEI – SHANGHAI	0.91	0.88	0.94		0.99	0.94	1.03	
HANG SENG – SHANGHAI	0.84	0.82	0.86		0.95	0.88	0.99	
Average	0.93	0.91	0.96	7.4%	0.97	0.92	1.02	3.3%
EUROPE - ASIA								
DAX - SHANGAI	0.94	0.92	0.97		0.98	0.94	1.03	
FTSE – SHANGHAI	0.94	0.92	0.97		0.99	0.95	1.03	
DAX – HANG SENG	0.85	0.83	0.88		0.95	0.91	0.99	
FTSE – HANG SENG	0.84	0.82	0.87		0.94	0.88	0.97	
FTSE – NIKKEI	0.81	0.79	0.83		0.96	0.92	0.99	
DAX - NIKKEI	0.77	0.75	0.79		0.93	0.89	0.97	
Average	0.86	0.84	0.89	8.1%	0.96	0.92	1.00	2.4%
EUROPE-US								
DAX – DOW JONES	0.83	0.81	0.86		0.95	0.91	0.99	
DAX – S&P500	0.82	0.79	0.84		0.92	0.86	0.97	
DOW JONES – FTSE	0.77	0.75	0.79		0.91	0.87	0.96	
FTSE – S&P500	0.76	0.74	0.78		0.89	0.86	0.94	
Average	0.80	0.77	0.82	4.4%	0.92	0.88	0.97	2.7%
US-ASIA								
DOW JONES – SHANGHAI	0.93	0.91	0.95		1.03	0.97	1.06	
SHANGHAI – S&P500	0.92	0.90	0.95		1.02	0.97	1.05	
DOW JONES – HANG SENG	0.79	0.77	0.81		0.96	0.91	0.99	
HANG SENG – S&P500	0.77	0.75	0.79		0.93	0.86	0.98	
DOW JONES – NIKKEI	0.75	0.73	0.77		0.95	0.91	0.99	
NIKKEI – S&P500	0.74	0.72	0.76		0.93	0.89	0.98	
Average	0.82	0.80	0.84	10.5%	0.97	0.92	1.01	4.6%
Overall	0.85	0.83	0.88	9.7%	0.96	0.91	1.00	3.8%

6. Concluding comments

The orders of integration in seven European, Asian and American stock market indices have been investigated in this work, firstly individually, and then by looking at the co-movements across the series in one-to-one differential relationships between them in the period from 2009 to 2020, comparing monthly, weekly and daily sampling periods.

The results obtained, based on fractional integration methods indicate that all the individual series are highly persistent with orders of integration very close to 1 in the majority of the cases, especially when allowing for autocorrelation errors. Evidence of mean reversion (i.e., estimates of d significantly below 1) is only obtained in some cases for the two American indices (S&P500 and Dow Jones) and generally, lower degrees of integration are obtained at lower (monthly) frequencies.

These results, showing little evidence of mean reversion are consistent with some other previous works (DePenya & Gil-Alana, 2004; Tabak, 2007; Narayan, 2008; Hasanov, 2009; Gozbasi et al., 2014; Tiwari & Kyophilavon, 2014; etc.), and the fact that there is a reduction in the magnitude of d at lower (monthly) frequencies is also consistent with Caporale et al. (2013) who show that lower degrees of integration were associated with lower frequencies.[3] Our results for the individual series differ from Adekoya (2021) who found evidence of mean reversion and long memory in 18 OECD countries using monthly samples and longer periods of analysis than in our work (in that paper the data spans from January, 1973 to August, 2018) and from Caporale et al. (2020a, 2020b) regarding the Russian market that found no-persistence or mean-reversion with daily samples between 2010 and 2018. These differences might be associated with the different sampling periods and the shock coverage. Other factors such as the globalization or the faster market behavior in recent times due to Internet and automatic computer orders could be brought into consideration when investigating these issues and should be analyzed in further research

Looking at the order of integration of the one-to-one differential series, we notice a reduction in the degree of integration, though the estimated values of d are relatively high. This implies that the long run equilibrium process is slow, and no evidence of standard cointegration across the series is apparent. The regional relationships with white noise calculations show evidence of stronger mean reversion, especially in the US-European markets but weaker in Asian-US and Asian-European directions, and very weak in the intra-region relationship at both white noise and autocorrelated errors. These results are in line with and complement those from Caporale et al. (2016), who analyzed the SP500 and the EuroStoxx50 for longer periods with a different cointegration methodology, demonstrating fractional cointegration at least for the subsample from December 1996 to March 2009, and ending when the global financial crisis was still severe. Other recent papers such as Lyócsa and Horváth (2018) or Budd (2018) agree in the transmission between markets. However, our study follows a different technique, showing evidence of weak mean reversion in the one-to-one differential comparisons, that also depends on the period of analysis and the sampling frequency used. Further research in this field should investigate other issues such as the potential presence of structural breaks and/or non-linearities in the data. This is relevant especially if we take into account that some authors have claimed that fractional integration may be a spurious phenomenon caused by the presence of breaks in the data (Diebold & Inoue, 2001; Ohanissian et al., 2008; Jin & Zhang, 2018; etc.) or even non-linearities (Granger & Hyung, 2004; Kuswanto & Sibbertsen, 2008; etc.). In this context, the linear time trend employed in this paper can be replaced by alternative non-linear trends using, for example, Chebyshev polynomials in time as

[3] The series examined in that paper were at high frequencies with a duration of one and a half days for the time period 13/05/2010 (11:47) – 14/05/2010 (21:07).

in Cuestas and Gil-Alana (2016), Fourier functions (Gil-Alana & Yaya, 2021) or neural networks (Yaya et al., 2021). Work in these directions is now in progress.

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