Monge, M., & Infante, J. (2022). A Fractional ARIMA (ARFIMA) Model in the Analysis of Historical Crude Oil Prices. *Energy RESEARCH LETTERS*, *4*(1). https://doi.org/10.46557/001c.36578

Peer-reviewed research

A Fractional ARIMA (ARFIMA) Model in the Analysis of Historical Crude Oil Prices

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Keywords: Crude oil prices, Fractional integration, persistence, JEL C00 C22 E30 Q40

https://doi.org/10.46557/001c.36578

Energy RESEARCH LETTERS

Vol. 4, Issue 1, 2023

We investigate historical data for crude oil prices using autoregressive fractionally integrated moving average (ARFIMA) models to determine whether shocks in the series have transitory or permanent effects. Our best specification is an ARFIMA(2,d,2) with an estimated value of d around 0.4, but its confidence interval is wide and does not allow us to either reject the I(0) or the I(1) hypotheses. This high level of uncertainty may be due to the presence of breaks or non-linear trends in the data.

I. Introduction

The objective of this paper is to evaluate the market persistence properties for annual historical crude oil prices from 1861 to 2019, studying its evolution across time. In particular, we examine the order of integration of the series, allowing this number to be a fractional value. The fractional integration or I(d) approach consists of taking d-differences in a given time series to render it stationary I(0), where d can be any real value, thus allowing for fractional degrees of differentiation. It permits us to distinguish between mean reversion and lack of it in a more flexible way than the standard methods that only use the values 0 (for stationary series) and 1 (for nonstationary ones). In the context of real values of d, mean reversion occurs as long as d is smaller than 1. The lower the value of d is, the faster the process of convergence to its original long term projection.

Earlier studies of fractional integration in the context of oil prices include, among others, Elder and Serletis (2008), Choi and Hammoudeh (2009), Gil-Alana and Yaya (2014), Gil-Alana et al. (2016), Gil-Alana, Yaya, and Awe (2017), Monge et al. (2017a, 2017b), Gil-Alana and Monge (2020), and Monge and Gil-Alana (2021).

It should be noted that that the AIC and BIC may not neccesarily be the best criteria in applications involving fractional integration (Beran et al., 1998; Hosking, 1984).

II. Data and Empirical Results

In order to carry out our analysis, data was drawn from *Our World in Data*, a project of the Global Change Data Lab, a non-profit organization based in the United Kingdom (Registered Charity Number 1186433). Historical crude oil prices analyzed are from 1861 to 2019 with annual data. *Our World in Data* is produced as a collaborative effort between researchers at the University of Oxford, who are the scientific contributors of the website content, and the non-profit organization, Global Change Data Lab, which owns, publishes and maintains the website and the data tools. At the University of Oxford, the research team is affiliated with the Oxford Martin Programme on Global Development, whose mission is to produce academic research on the world's largest problems based on empirical analysis of global data.

Using this dataset, we first conduct standard unit root tests (Dickey & Fuller, 1979; Kwiatkowski et al., 1992; Phillips & Perron, 1988) on the crude oil prices. According to the results, as presented in Table 1, the time series analyzed are clearly non-stationary I(1). The results are available from the authors upon request.

Nevertheless, authors such as Diebold and Rudebusch (1991), Hassler and Wolters (1994) and Lee and Schmidt (1996) show that the unit root methods have low power under fractional alternatives. For this reason, we use the ARFIMA (p, d, q) model where the mathematical notation is:

$$\Phi(L)(1-L)^d X_t = \Theta(L)u_t, \qquad t = 1,\ 2,\ \ldots, \quad (1)$$

where the time-series x_t, $t = 1,\ 2,\ \ldots$ follows an integrated order process d (and is denoted as $x_t \approx I(d)$) and where d refers to any real value, L refers to the lag-operator ($Lx_t = x_{t-1}$) and u_t is ARMA(p, q) such that ε_t is a white noise process. The Akaike information criterion (Akaike, 1973) and Bayesian information criterion (Akaike, 1979) were used to select the appropriate AR and MA orders in the models.

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Table 1. Unit root test results

	ADF			PP		KPSS	
	(i)	(ii)	(iii)	(ii)	(iii)	(ii)	(iii)
Oil prices	-0.9827	-1.4693	-2.5776	-1.1455	-2.2716	1.6264	0.4792

Notes: This table reports the unit root test results. (i) Refers to the model with no deterministic components; (ii) with an intercept, and (iii) with a linear time trend. I reflect t-statistic with test critical value at 5%.

Table 2. Results of long memory tests

Data Analyzed	ARFIMA model	d	Std. Error	Interval	I(d)
	ARFIMA (0, d, 0)	0.9506	0.0757	[0.83, 1.07]	I(1)
	ARFIMA (1, d, 0)	0.6858	0.1556	[0.43, 0.94]	I(d)
	ARFIMA (2, d, 0)	0.7777	0.1658	[0.50, 1.05]	I(1)
	ARFIMA (0, d, 1)	0.7325	0.0872	[0.59, 0.88]	I(d)
Oil prices	ARFIMA (0, d, 2)	0.7790	0.1323	[0.56, 1.00]	I(1)
	ARFIMA (1, d, 1)	0.7817	0.1065	[0.61, 0,96]	I(d)
	ARFIMA (2, d, 1)	0.7604	0.1543	[0.51, 1.01]	I(1)
	ARFIMA (1, d, 2)	0.7754	0.1167	[0.58, 0.97]	I(d)
	ARFIMA (2, d, 2)	**0.4085**	**0.6464**	**[-0.65, 1.47]**	**I(0), I(d), I(1)**

Notes: The table reports the long memory test results. In bold we have selected the ARFIMA (2, d, 2) model following the criteria (greater value) of AIC and BIC.

The d parameter has been estimated considering all combinations of AR and MA terms ($p, q \leq 2$) for each time-series, with their confidence bands at 95%. The results have been displayed in Table 2.

We observe that the estimates of d are close to 0.70 in almost all cases. The exceptions are the ARFIMA (0, d, 0) with a value of d close to 1 (0.95), and where the unit root null hypothesis cannot be rejected and the ARFIMA (2, d, 2) model is precisely the one selected with the AIC and BIC mentioned above, and that gives an estimator of $d = 0.40$. It may then be concluded that the time series is mean reverting where the order of integration is smaller than 1, however, we also observe that the standard error in this case is very large, implying that based on the wide confidence bands, we are not able to reject the hypotheses of I(0) and I(1) behaviour. This fact may occur due to the presence of structural changes throughout the period examined.

III. Conclusion

The orders of integration in annual historical crude oil prices have been investigated in this work for the time period of 1861 to 2019. Our goal is to demonstrate whether the results, based on fractional integration methods, show evidence of persistence or mean reversion. Based on likelihood criteria, it is found that the estimated value of d is around 0.40 and thus shows mean reversion, but the confidence band is so wide that the I(0) and I(1) hypotheses cannot be rejected.

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